Solutions – excercise 1

1) $\frac{S(t)}{S(0)} = e^{Y}$ Y Gaussian with mean μt, variance $\sigma^2 t$

lectures: $G_Y(K) = \langle e^{iKY} \rangle = e^{iK\mu t - \frac{1}{2}\sigma^2 t K^2}$

$$\Rightarrow \langle S(t)^m \rangle = (S(0))^m \langle e^{mY} \rangle = (S(0))^m \, G_Y\left(\tfrac{m}{i}\right)$$
$$= (S(0))^m \, e^{m\mu t + \frac{1}{2}\sigma^2 t m^2}$$

variance: $\langle S(t)^2 \rangle - \langle S(t) \rangle^2 = (S(0))^2 \left(e^{2\mu t + 2\sigma^2 t} - e^{2(\mu t + \frac{1}{2}\sigma^2 t)}\right)$
$$= (S(0))^2 e^{2\mu t}\left(e^{2\sigma^2 t} - e^{\sigma^2 t}\right)$$

2) $$P \cdot \left(1 + \frac{r_m}{m}\right)^m = P\left(1 + \frac{r_n}{n}\right)^n$$

$$\Rightarrow \left(1 + \frac{r_m}{m}\right)^{\frac{m}{n}} = 1 + \frac{r_n}{n} \Rightarrow r_n = n\left\{\left(1 + \frac{r_m}{m}\right)^{\frac{m}{n}} - 1\right\}$$

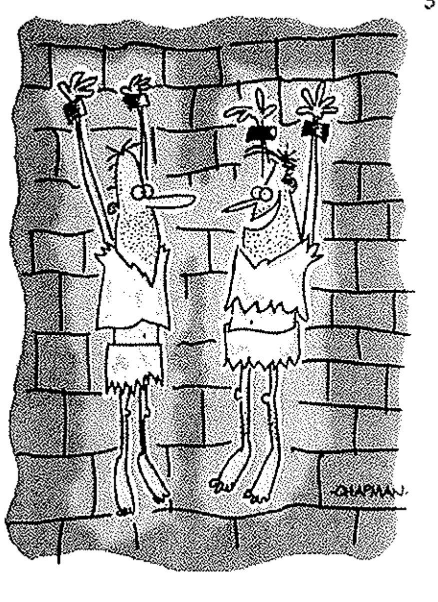
3) $$a_1\beta + a_2\beta^2 + a_3\beta^3 = b_1\beta + b_2\beta^2 + b_3\beta^3 \quad |:\beta$$
where $\beta := \frac{1}{1+r}$

$$\Rightarrow (a_1 - b_1) + (a_2 - b_2)\beta + (a_3 - b_3)\beta^2 = 0$$

$$\Leftrightarrow \beta^2 + \frac{a_2 - b_2}{a_3 - b_3}\beta + \frac{a_1 - b_1}{a_3 - b_3} = 0$$

$$\beta_{1/2} = -\frac{1}{2}\frac{a_2 - b_2}{a_3 - b_3} \pm \sqrt{\left(\frac{a_2 - b_2}{a_3 - b_3}\right)^2 \frac{1}{4} - \frac{a_1 - b_1}{a_3 - b_3}}$$

$$r_{1/2} = \frac{1}{\beta_{1/2}} - 1$$



ON THE BRIGHT SIDE... WE'RE NOT
LIVING PAYCHECK TO PAYCHECK ANYMORE.

4*) a) lectures: $r_{eff} = (1+\frac{r}{n})^n - 1$

continuous interest means $n \to \infty$

$\Rightarrow$ $r_{eff} = \lim_{n\to\infty} (1+\frac{r}{n})^n - 1 = e^r - 1$ (20)

b) lectures: $D(t) = D(0) \cdot \exp\{ \int_0^t r(\tau)\,d\tau \}$

Hence choose that bank for which $\int_0^t r(\tau)\,d\tau$ has a maximum.

$$\int_0^t r_A(\tau)\,d\tau = \frac{1}{100}\int_0^t (1+\sin\pi\tau)\,d\tau = \frac{1}{100}\cdot\left(t - \frac{1}{\pi}\cos\pi\tau\Big|_0^1\right)$$

$$= \frac{1}{100}\left(t - \frac{1}{\pi}(-1-1)\right) = \frac{1}{100}\left(t+\frac{2}{\pi}\right)$$

$$= 0.01637 \quad \text{for } t = 1 \text{ year} \quad (10)$$

$$\int_0^t r_B(\tau)\,d\tau = \frac{1}{70}\int_0^t e^{\frac{1}{10}\tau}\,d\tau = \frac{1}{70}\,\frac{e^{\frac{1}{10}\tau}}{\frac{1}{10}}\Bigg|_0^t = \frac{1}{7}\left(e^{\frac{1}{10}t} - 1\right)$$

$$= 0.0150 \quad \text{for } t = 1 \text{ year} \quad (10)$$

$$\int_0^t r_C(\tau)\,d\tau = \frac{1}{68}\int_0^t d\tau = \frac{1}{68}t = 0.0147 \quad \text{for } t = 1 \text{ year} \quad (10)$$

$\Rightarrow$ choice A is best. (10)

Investing $D(0) = £\,1000$ you have at $t=1$

$$D(1) = D(0)\cdot e^{0.01637} = £\,1016.50 \quad (10)$$

c) Government wants to stimulate economy by setting low interest rate. By this, borrowing money is much cheaper, so investments are easier to do, which should lead out of the current crisis and lead to more economic activity. (30)