

Solutions - exercise 3

1) $\langle M(t)^m \rangle = (M(0))^m e^{m\mu t + \frac{1}{2}\sigma^2 t m^2}$ (Q1, coursework 1)

$t < t_D \Rightarrow S(t) = M(t) \Rightarrow \langle S(t)^m \rangle = (S(0))^m e^{m\mu t + \frac{1}{2}\sigma^2 t m^2}$

$t \geq t_D \Rightarrow S(t) = (1-f)M(t) \Rightarrow \langle S(t)^m \rangle = (S(0))^m (1-f)^m e^{m\mu t + \frac{1}{2}\sigma^2 t m^2}$

2) lectures: $\frac{\partial C}{\partial K} = -e^{-rt} \phi(\omega - \sigma\sqrt{t})$

thus $\frac{\partial^2 C}{\partial K^2} = -e^{-rt} \phi'(\omega - \sigma\sqrt{t}) \cdot \frac{\partial \omega}{\partial K}$

$\omega = \frac{rt + \frac{1}{2}\sigma^2 t - \log K + \log S}{\sigma\sqrt{t}} \Rightarrow \frac{\partial \omega}{\partial K} = -\frac{1}{\sigma\sqrt{t}} \frac{1}{K}$

$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{1}{2}y^2} dy \Rightarrow \phi'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

$\Rightarrow \frac{\partial^2 C}{\partial K^2} = \underbrace{e^{-rt}}_{>0} \underbrace{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\omega - \sigma\sqrt{t})^2}}_{>0} \underbrace{\frac{1}{K\sigma\sqrt{t}}}_{>0} > 0 \Rightarrow \text{convex}$

3) $\left. \begin{aligned} S + P_1 - C_1 &= Ke^{-rt} \\ S + P_2 - C_2 &= Ke^{-2t} \end{aligned} \right\} \text{subtract} \Rightarrow P_1 - C_1 - P_2 + C_2 = K(e^{-rt} - e^{-2t})$

(put-call option parity formula, see lectures)

Let $x := e^{-t} \Rightarrow \frac{P_1 - C_1 - P_2 + C_2}{K} = x - x^2$

$\Leftrightarrow x^2 - x + \frac{1}{K}(P_1 - C_1 - P_2 + C_2) = 0$

$x_{1/2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{K}(P_1 - C_1 - P_2 + C_2)} = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{2}{100}} = \frac{1}{2} \pm \sqrt{\frac{23}{100}}$

$\Rightarrow r = -\log x = -\log\left(\frac{1}{2} + \sqrt{\frac{23}{100}}\right) = 0.0206 = 2.06\%$

↑ reasonable interest rate for + sign

4) e.g. August 2007

closing prices (in pence)	3235.13	3258.33	3223.84	3200.15	...
	1 Aug	2 Aug	3 Aug	6 Aug	

$\Rightarrow 20$ data points $X_1 = 0.00715$ $X_2 = -0.01064$ $X_3 = -0.00738$...

$\bar{X} = \frac{1}{20} \sum_{i=1}^{20} X_i = 0.00039$

$\sigma = \sqrt{\frac{1}{2} \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2} \approx 0.31$

$l = \frac{t}{n}$ $t \approx \frac{1}{12}$ $n = 20$

$\mu = \frac{1}{l} \bar{X} = 0.094$ $MSE(\hat{\sigma}^2) = \text{var} \hat{\sigma}^2 \approx \frac{2}{n-1} \sigma^4 \approx 1.10$

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David E won
£629,548
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PRICES

Date	Open	High	Low	Close	Volume	Adj Close*
31-Aug-07	3,211.58	3,262.05	3,211.58	3,260.48	0	3,260.48
30-Aug-07	3,168.18	3,215.10	3,166.52	3,211.58	0	3,211.58
29-Aug-07	3,151.43	3,170.80	3,127.60	3,168.18	0	3,168.18
28-Aug-07	3,209.46	3,209.46	3,143.74	3,151.43	0	3,151.43
27-Aug-07	3,198.06	3,215.60	3,190.83	3,209.46	0	3,209.46
23-Aug-07	3,195.66	3,245.13	3,195.66	3,198.06	0	3,198.06
22-Aug-07	3,136.36	3,196.34	3,136.36	3,195.66	0	3,195.66
21-Aug-07	3,136.05	3,155.63	3,113.32	3,136.36	0	3,136.36
20-Aug-07	3,128.97	3,177.96	3,128.97	3,136.04	0	3,136.04
17-Aug-07	3,031.91	3,160.75	3,014.01	3,128.97	0	3,128.97
16-Aug-07	3,159.24	3,159.24	3,031.91	3,031.91	0	3,031.91
15-Aug-07	3,179.17	3,179.17	3,127.80	3,159.24	0	3,159.24
13-Aug-07	3,129.62	3,226.14	3,129.62	3,219.05	0	3,219.05
10-Aug-07	3,244.62	3,244.62	3,128.72	3,129.62	0	3,129.62
9-Aug-07	3,307.94	3,307.94	3,222.84	3,244.62	0	3,244.62
8-Aug-07	3,261.71	3,313.96	3,261.71	3,307.93	0	3,307.93

(Advertisement)

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Example

August 2007

$$X_i^e = \log \frac{\text{new closing price}}{\text{old closing price}}$$

- $X_{20}^e = 0.01511$
- $X_{19}^e = 0.01361$
- $X_{18}^e = 0.00530$
- $X_{17}^e = -0.01825$
- $X_{16}^e = 0.00356$
- $X_{15}^e = 0.00075$
- $X_{14}^e = 0.01873$
- $X_{13}^e = 0.00010$
- $X_{12}^e = 0.00226$
- $X_{11}^e = 0.03151$
- $X_{10}^e = -0.09114$
- $X_9^e = -0.01875$
- $X_8^e = 0.02817$
- $X_7^e = -0.03603$
- $X_6^e = -0.01932$
- $X_5^e = 0.01407$

31 Aug				3260.48		
7-Aug-07	3,200.15	3,261.72	3,200.15	3,261.72	0	3,261.72
6-Aug-07	3,223.84	3,228.25	3,190.21	3,200.15	0	3,200.15
3-Aug-07	3,258.33	3,270.38	3,217.21	3,223.84	0	3,223.84
2-Aug-07	3,235.13	3,266.87	3,235.13	3,258.33	0	3,258.33
1-Aug-07	3,289.12	3,289.12	3,202.45	3,235.13	0	3,235.13

$$X_4 = 0.01906$$

$$X_3 = -0.00738$$

$$X_2 = -0.01064$$

$$X_1 = 0.00715$$

* Close price adjusted for dividends and splits.

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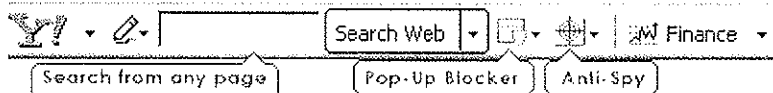
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$$\bar{X} = \frac{1}{20} \sum_{i=1}^{20} X_i = 0.00039$$

$$\text{sample variance} = \frac{1}{19} \sum_{i=1}^{20} (X_i - \bar{X})^2 = 4.02 \cdot 10^{-4}$$

$$l = \frac{t}{n} \quad t \approx \frac{1}{12} \quad (1 \text{ month})$$

$$\Rightarrow l \approx 0.00417$$

$n = 20$
 ↑
 number of data points used

$$\mu = \frac{1}{l} \bar{X} = 0.094$$

$$\sigma^2 \approx \frac{1}{l} \text{sample variance} = 0.0964 \Rightarrow \sigma \approx 0.31$$

$$\text{MSE}(\hat{\sigma}^2) = \text{var } \hat{\sigma}^2 \approx \frac{2}{n-1} \sigma^4 \approx 1 \cdot 10^{-3}$$

↑
lecturer

This is an example only.

The numbers $\mu, \sigma, \text{var}(\hat{\sigma}^2)$ may strongly fluctuate for the different months.