Exercise Sheet 5
MTH6120 Further Topics in Mathematical Finance

due: Monday, 27 February 2012, 1.15pm

1. Are the following utility functions strictly concave?

   i) $u(x) = 17 + x + \log(x + a)$

   ii) $u(x) = 3 - e^{-bx}$

   iii) $u(x) = \frac{1-x^{1-q}}{q-1}$

   Give reasons. Distinguish, if necessary, between positive, negative, and vanishing values of the parameters a, b, q.

2. Consider a risk-neutral share price evolution model based on geometric Brownian motion and lognormally distributed jumps with the parameters $\sigma^2 = \mu = \sigma_0^2 = \mu_0 = \frac{1}{10}$. Write down a formula for the no-arbitrage cost C_J of a call option as a function of the parameters s, t, K and λ. Approximate your formula by taking into account only the first two terms in the summation. Derive an approximate relation between C_J and the option cost given by the Black-Scholes formula if $s = 1$, $K = 2$, $t = 1$ and $\lambda = \frac{1}{100}$.

3. Prove that if the expectation $\langle J \rangle$ of jump factors is bigger than 1, then the no-arbitrage cost C_J of a call option in a model with jumps satisfies $C_J(s, t, K, \sigma, r) \geq C(s, t, K, \sigma, \tilde{r})$, where C is the usual option price as given by the Black-Scholes formula and $\tilde{r} := r + \lambda(1 - \langle J \rangle)$.

   Hint: You may use the fact that $\frac{\partial}{\partial x} C \geq 0$, where $x = t, \sigma, r$.

4. Consider a general 3-state model where the share price $S(0)$ can evolve to 3 values s_1, s_2, s_3 at $t = 1$, where $s_1 > s_2 = S(0) > s_3$. The interest rate is $r = 0$ and the strike price is K, with $s_1 > K > s_2$. Assuming that there are no arbitrage opportunities, determine the allowed range of option prices C.