Queen Mary
University of London

B. Sc. Examination by course unit 2010

MTH6120 Further Topics in Mathematical Finance

Duration: 2 hours

Date and time: 26 May 2010, 14:30

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): Prof. C. Beck

1.

(a) [3 marks] Explain what is an arbitrage opportunity.

(b) [5 marks] Define what is geometric Brownian motion $S(t)$. What condition is satisfied by risk-neutral geometric Brownian motion?

(c) [6 marks] Calculate the moments of risk-neutral geometric Brownian motion.

Hint: You may use the fact that the characteristic function of a Gaussian distribution with mean μ_0 and variance σ_0^2 is $G(k) = e^{ik\mu_0 - \frac{1}{2}\sigma_0^2 k^2}$.

(d) [5 marks] Suppose a company pays a dividend at a fraction f of the share price at time t_D. Explain how the share price $S(t)$ changes at t_D.

(e) [6 marks] Calculate the variance of the share price $S(t)$ after a divident payment at fixed time $t_D < t$ has been made, assuming that the market value of the share obeys geometric Brownian motion.

2.

(a) [5 marks] Explain what is a European call option and what is a European put option. What is the difference between European, American, and Asian options?

(b) [3 marks] State the put-call option parity formula.

(c) [6 marks] Suppose you know the prices of put options and call options of the same share with the same strike price K for two different expiration times, $t_1 = 1$ year and $t_2 =$ half a year. Derive from this information the possible interest rates r. Is the result unique?

(d) [5 marks] State the main result of the Black-Scholes model, the Black-Scholes option pricing formula for the cost C of a call option. Explain the meaning of the parameters s, t, K, σ, r and ω.

(e) [6 marks] A bank pays interest continuously with time-dependent rate function $r(t) = \frac{1}{1+t} + \log 1.9$. Calculate what is in your account after one year if you initially deposit £1000.

3.

(a) [4 marks] Briefly explain what is the Poisson process and what it is used for in share price evolution models.

(b) [6 marks] Consider a Poisson process for share price jumps, where the probability to observe $N(t) = n$ jumps in the time interval $[0, t]$ is given by $p_t(n) = e^{-\lambda t}\frac{(\lambda t)^n}{n!}$. Calculate

i) $\langle N \rangle$

ii) $\langle e^{2N} \rangle$

(c) [7 marks] Determine the value of t where $p_t(n)$ takes on a maximum. Prove that $\max_t\{p_t(n)\}$ is independent of λ.

(d) [8 marks] Consider a share price model of the form $S(t) = S^*(t)\prod_{i=1}^{N(t)} J_i = S^*(t)J(t)$, where the J_i are independent jump factors with average $\langle J \rangle$ and $S^*(t)$ is geometric Brownian motion with mean parameter μ and variance parameter σ^2. $N(t)$ is a Poisson process of intensity λ. Calculate

i) the conditional expectation $E(J(t)|N(t) = n)$.

ii) the expectation $\langle S(t) \rangle$.

Your formulas should only depend on $\langle J \rangle, \lambda, \mu$ and σ^2.

4.

(a) [4 marks] Explain what is a utility function.

Is the following utility function strictly concave? $u(x) = a + \log(b + x)$

(a, b are constants).

(b) [7 marks] An investor with capital x can invest any amount αx, where $0 \le \alpha \le 1$. If the amount αx is invested she may either receive $7\alpha x$ with probability p or 0 with probability $1 - p$. No interest is paid. The investor's utility function is $u(x) = 2 + \log x$. Given p, how much money should she invest?

(c) [4 marks] What is a portfolio? What general condition characterizes the optimal portfolio for a given investor?

(d) [5 marks] Four different share prices evolve independently. The rates of return have the same mean values but different variances $v_i^2 = \frac{1}{i+1}$, $i = 1, 2, 3, 4$. You have £28000 to invest. Construct the optimal portfolio.

(e) [5 marks] Briefly explain what is the Knapsack problem.

End of Paper