Exercise Sheet 7
MTH6120 Further Topics in Mathematical Finance

due: Wednesday, 17 March 2010, 10am

1. Three different share prices $S_i(t)$, $i = 1, 2, 3$ evolve independently according to risk-neutral geometric Brownian. The variance parameters are $\sigma_1^2 = 0.1$, $\sigma_2^2 = 0.2$, $\sigma_3^2 = 0.5$ respectively. You have $w =$ *£10000* to invest into these shares for a period of 1 year. Construct the optimal portfolio.

   Hint: Use the results of Q2 on exercise sheet 6.

2. Consider the CAPM model with $r = 0$. Calculate $var(\xi_i)$. That is to say, derive a formula for the variance of the random variable ξ_i that depends on v_i, v_m and β_i only.

3. Find on the internet the beta values of the following shares: i) GlaxoSmithKline (GSK) ii) BP iii) Rio Tinto (RIO). Do you have any economic explanation for the relative size of the β values of these three companies?

4. *

   (same as Q1 on exercise sheet 6, but with different utility function)

   An investor with capital x can invest any amount αx, where $0 \leq \alpha \leq 1$. If the amount αx is invested he may either receive $3\alpha x$ with probability p or 0 with probability $1 - p$. No interest is paid. The investor's utility function is $u(x) = \sqrt{x}$. How much money should be invested? Determine the amount to be invested for the special case that $x =$ *£1000* and that

   i) $p = \frac{1}{7}$

   ii)$p = \frac{1}{3}$

   iii)$p = \frac{2}{3}$

   iv) $p = 1$.