Exercise Sheet 6
MTH6120 Further Topics in Mathematical Finance

due: Wednesday, 10 March 2010, 10am

1. An investor with capital x can invest any amount αx, where $0 \leq \alpha \leq 1$. If the amount αx is invested he may either receive $3\alpha x$ with probability p or 0 with probability $1-p$. The investor's utility function is $u(x) = \log x$. No interest is paid. How much money should be invested?

2. Let share prices $S_i(t)$, $i = 1, \ldots, n$ evolve independently according to geometric Brownian motion with parameters μ_i and σ_i. Let the period under consideration be t. Determine i) the rate of return R_i, ii) r_i, iii) v_i^2, iv) $cov(R_i, R_j)$ $(i \neq j)$, v) $var(\log(R_i + 1))$.

3. Estimate the mean and the variance of the various guesses we had in our class for the future value of the FTSE100. Which person's guess is closest to the mean? Is your own personal guess i) above or below the mean ii) within one standard deviation iii) within 3 standard deviations?

4. *

   i) Consider two securities with different expectations of the rate of return r_i $(i = 1, 2)$, and different variances v_i^2 $(i = 1, 2)$. The utility function is $u(x) = 1 - e^{-bx}$. Find the general formula for the optimal portfolio (w_1, w_2).

   ii) Prove that this is really the optimum portfolio and not the worst possible. In other words, show that the expected utility has a maximum, rather than a minimum.

   iii) You want to investe your savings of *£6500* into shares of two different companies. The first stock has a rather small expectation of rate of return $r_1 = 0.1$ and a rather small variance $v_1^2 = 0.1$, whereas the second one has a larger expectation of rate of return $r_2 = 0.2$, but is also more risky since it has larger variance $v_2^2 = 0.5$. The correlation coefficient between both shares is $\rho = 0.1$ Your utility function is $u(x) = 1 - e^{-bx}$, $b = 0.001$. How much should you invest into each stock?