Exercise Sheet 4
MTH6120 Further Topics in Mathematical Finance

due: Wednesday, 17 February 2010, 10am

1. Prove the following statement: If either all individual share options end in the money or none of them does, then the payoff of a call option on a share price index is equal to the sum of payoffs from the options on the individual shares in the portfolio.

2. Consider a European (K, t) call option whose return at expiration time t is capped by the amount $B > 0$. That is, the payoff at time t is given by $\min((S(t) - K)^+, B)$. Use the Black-Scholes theory to find the no-arbitrage cost of this option.

   Hint: Express the payoff in terms of the payoffs from two plain (uncapped) European call options.

3. (Valentine's question) In a pub you meet a person (Mr. X) who turns out to possess options on BP shares, just as you do! Even the strike price of *£9* and the expiration time of half a year is the same! The only difference is that he possesses 100 call options, whereas you possess 100 put options. Mr. X invites you for a free dinner on Valentine's day in an expensive London restaurant (worth *£60*), and he suggests that on this occasion – as a symbol of deep friendship – he will give you half of his call options whereas you give him half of your put options. Should you accept?

4. * Consider a Poisson process for share price jumps, where the probability $p_t(n)$ to observe $N(t) = n$ jumps in the time interval $[0, t]$ is given by $p_t(n) = e^{-\lambda t}\frac{(\lambda t)^n}{n!}$. Calculate

   a) $\langle N \rangle$

   b) $\langle N(N-1) \rangle$

   c) $\langle e^N \rangle$

   d) $\langle N^2 \rangle - \langle N \rangle^2$

   e) $\max_t \; p_t(n)$