Exercise Sheet 3
MTH6120 Further Topics in Mathematical Finance

due: Wednesday, 10 February 2010, 10am

1. At a given time t_D a company pays a dividend to its share holders at a fraction f of the share price. Assuming that the market value $M(t)$ of the share obeys geometric Brownian motion, calculate the moments of the share price $S(t)$ before and after the dividend payment.

2. From the lectures we know that the cost of a call option $C(K,t)$ is always a convex function of the strike price K, no matter what models the share price evolution. Calculate $\frac{\partial^2}{\partial K^2}C$ explicitly for the example of the Black-Scholes theory, and show that in this case the second derivative is indeed always positive.

3. We are in the year 3000. During an archaeological expedition to the former city of London, a little piece of the Financial Times is found, dated 19 February 2020. It states that for BP shares the price of a put option with strike price $K = £100.00$ is $P_1 = £23.00$ if the expiration time is 1 year and $P_2 = £26.00$ if the expiration time is 2 years. Moreover, the price of a BP call option with the same strike price is $C_1 = £19.00$ for 1 year expiration time and $C_2 = £24.00$ for 2 years. The archaeologists want to know what the UK interest rate was during February 2020. Please help them.

4. *

   (a) State your birthday.

   (b) We want to analyse real share price data. Choose as the relevant month your birthday month. Choose as the relevant year the year 2008 or 2010 if you are female and the year 2007 or 2009 if you are male. Find on the internet the daily closing prices of BHP Billiton shares (symbol: BLT.L) at the London Stock exchange during your birthday month, and print this list.

   (c) Calculate the random variables $X_i = \log \frac{S(il)}{S((i-1)l)}$ for your data set.

   (d) Estimate the mean parameter μ and the volatility parameter σ from your data set. Also, provide a rough estimate of the mean square error (MSE) of your determined value σ^2.