

Exercise Sheet 2

MTH6120 Further Topics in Mathematical Finance

due: Wednesday, 3 February 2010, 10am

1. An exotic betting shop pays you for each unit of money you bet the amount o_i^2 if your bet is correct, otherwise it charges you three times the amount you have betted. Assuming there are no arbitrage opportunities, derive the set of probabilities p_i of the events i from these return functions.
2. Consider the Black-Scholes model where the cost of a call option is given by $C(s, t, K, \sigma, r) = e^{-rt} \langle (S(t) - K)^+ \rangle$. Prove that

$$\frac{\partial C}{\partial r} = Kte^{-rt}\Phi(\omega - \sigma\sqrt{t}).$$

3. Prove Lemma 2 of the lectures using Lemma 1.
4. *

(a) A company continuously pays a dividend to its shareholders with the rate $f = 0.03$. Currently one share is worth $\pounds 40.40$. The volatility parameter is $\sigma = 0.24$ and the nominal interest rate is 4%. You want to buy a call option with a strike price of $K = \pounds 42.00$ and an expiration time of 4 months. What will you have to pay for this option? What is the probability that your option finishes in the money?

The above company wants to switch from paying continuous dividends at rate f to paying a dividend every 4 months at rate f_{new} . What is the relation between f_{new} and f that keeps the option price invariant?

(b) Find on the internet the evolution of the *FTSE100* over the past 5 years. Print this curve. Describe the main features of this curve, with the emphasis on major changes. Do you observe pure random behaviour or are there economic reasons for what you observe?

(c) Guess what the *FTSE100* will be on 25 March 2010 at the closing time of the London stock exchange (4.30 pm). Write this 4-digit number on the top page of your coursework submission. The person with the closest guess will win a $\pounds 20$ cash prize.