MTH5122 Statistical Methods — Tutorial 7

You should attempt all of these questions, as they are designed to help you to learn and understand the material in the course.

The Feedback question is the one for handing in. Write your name and student number and the time of the tutorial you are allocated to (for example, Tuesday, 3 - 4 pm) at the top of your answer sheet. Staple all the pages together and put them into the ***blue box*** *in the basement of the Mathematics Building by* ***4 pm on Wednesday 24 November 2010****.*

If you want help on any of the other questions, or want to check that you have done them correctly, you may ask any helper during your tutorial session.

Question 1 Let $\boldsymbol{Y} = (Y_1, \dots, Y_n)$ be a random sample from the distribution with the pdf given by

$$f_Y(y) = \begin{cases} \frac{2}{\vartheta^2}(\vartheta - y), & y \in [0, \vartheta], \\ 0, & \text{elsewhere.} \end{cases}$$

For an estimator $T(\boldsymbol{Y}) = 3\overline{Y}$ calculate its bias and variance and check if it is a consistent estimator for the parameter ϑ.

Question 2 Let $X \sim \text{Poisson}(\lambda)$, where $\lambda = 9$. Denote: $\text{E}(X) = \mu$ and $\text{var}(X) = \sigma^2$. Use the New Cambridge Elementary Statistical Tables to find the probabilities $P(|X - \mu| \geq k\sigma)$ for $k = 1, 2, 3$. Comment on the results.

Question 3 Let $X \sim \chi^2_\nu$, where $\nu = 2$. Denote: $\text{E}(X) = \mu$ and $\text{var}(X) = \sigma^2$. Use the New Cambridge Elementary Statistical Tables to find the probabilities $P(|X - \mu| \geq k\sigma)$ for $k = 1, 2, 3$. Comment on the results.

Feedback Question 1 Let $X_i \underset{iid}{\sim} \text{Bernoulli}(p)$. Denote by $\overline{X}$ the sample mean.

(a) Show that $\widehat{p} = \overline{X}$ is a consistent estimator of p.

(b) Show that $\widehat{pq} = \overline{X}(1 - \overline{X})$ is an asymptotically unbiased estimator of pq, where $q = 1 - p$.

Feedback Question 2 Let $X_1 \sim t_{\nu_1}$, where $\nu_1 = 3$ and let $X_2 \sim t_{\nu_2}$, where $\nu_2 = 30$. Denote: $\text{E}(X_i) = \mu_i$ and $\text{var}(X_i) = \sigma_i^2$ for $i = 1, 2$. Use the New Cambridge Elementary Statistical Tables to find the probabilities $P(|X - \mu_i| \geq k\sigma_i)$ for $i = 1, 2$ and $k = 1, 2, 3$. Comment on the results. How does the probability of $P(|X - \mu_2| \geq 3\sigma_2)$ compare with the respective probability for a standard normal distribution?