Dynamical modelling of TCP packet traffic on scale-free networks

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Abstract

The interactive growth method is used to model the topology of real networks. Packet traffic is simulated crossing this network using the closed-loop packet transfer mechanism *Transmission Control Protocol*. Comparisons are made for traffic on regular and scale-free networks with open-loop and closed-loop packet transfer mechanisms. Packet lifetimes and queue behaviour for long range dependent sources (LRD) are compared with short range dependent Poisson sources (SRD) at the same loadings. The effects of varying server strengths are studied as are the results of imposing packet loss. The robustness of results is tested by varying patterns of hosts and using different networks with similar parameters. A marked difference is seen between outputs from the two source types, SRD and LRD, emphasizing that long range dependence in sources is an important factor. Changing host patterns for interactive growth networks produces very similar results indicating a good degree of robustness in the simulations. However, these results are very different from those obtained for regular and scale-free network simulations using an open-loop transfer mechanism. This demonstrates the need for more accurate models such as the interactive growth model, and for the simulation of closed-loop algorithms such as transmission control protocol.

1 Introduction

Over the past few years many studies have been made of the internet. Measurements and simulations have revealed the long range dependent nature of packet traffic on the internet, [?]. In parallel with this work the topology of the internet has been seen to be a scale-free structure [?, ?, ?]. In this paper, these two aspects of the internet have been incorporated into one simulation. The intention of these changes is to model the real internet much more accurately.

Long range dependence (LRD) in internet traffic was first demonstrated by [?]. The *strength* of LRD can be characterised by the Hurst parameter, H, (see [?]) which varies between 0.5 and 1. The value $H = 0.5$ is equivalent to a traffic trace having no LRD (thus, it is short range dependent, SRD); the value $H = 1$ represents the maximum level of long range dependence. Fig. 1 illustrates the difference between LRD and SRD sources of traffic. Batch averages have been taken from binary data representing packets coming from a source. The plot in Fig. 1(b) shows variances in these batch average values measured for runs of 100,000 batches. A range of batch sizes and H values have been used. Comparing $H = 0.5$ (SRD) with $H = 0.974$ (LRD), it can be seen that the decay in variance with increasing batch size is much less at the higher H value. This illustrates an important property of LRD traffic. Even taking very large batches, the mean value can still vary considerably between batches. This is linked to the bursty nature of LRD traffic as depicted in Fig. 1(a). Very long sequences of 1's or 0's are possible, so even very long runs cannot be relied upon to produce a predictable average number of packets. Poisson sources are short range dependent and therefore do not exhibit this bursty behaviour.

The second main characteristic of LRD traffic is that it is self-similar. This can be illustrated by measuring packet frequencies over different time scales (Fig. 1(c)). It can be seen that changes in time scale do not change the general appearance of the traffic. In Fig. 1(d), a plot of the auto-correlation function for LRD traffic shows that this self-similarity results in a power law decay with increasing lag. At a Hurst parameter of 0.97 a very slow power law decay is seen. At an H value of 0.5 the source is short range dependent and the autocorrelation decays much more rapidly. The slope of this log-log plot, given by β, is related to the Hurst parameter by the formula:

$$H = 1 - \beta/2. \qquad (1)$$

In earlier work, we simulated packet traffic in regular networks, [?, ?], using an open-loop packet transport mechanism. Networks using LRD and Poisson sources were compared. The *transmission control protocol* (TCP) [?] is the dominant packet transfer mechanism in today's internet. TCP is favoured in the internet because most data is sent from one host to another, and needs to be sent without error. TCP requires all packets sent to be acknowledged by the receiver before further packets can be dispatched. In this way packets are only sent when a connection between sender and receiver is established. Any packets not acknowledged by the receiver are sent again. This mechanism provides a very reliable method for data transfer.

Since TCP is a closed-loop protocol we would expect behaviour different to that seen with the open-loop simulations. Erramilli *et al.*, (2002),[?], have already shown that TCP feedback modifies self-similarity in offered traffic without creating or removing the self-similarity itself. In this paper we use a simplified model of the most commonly

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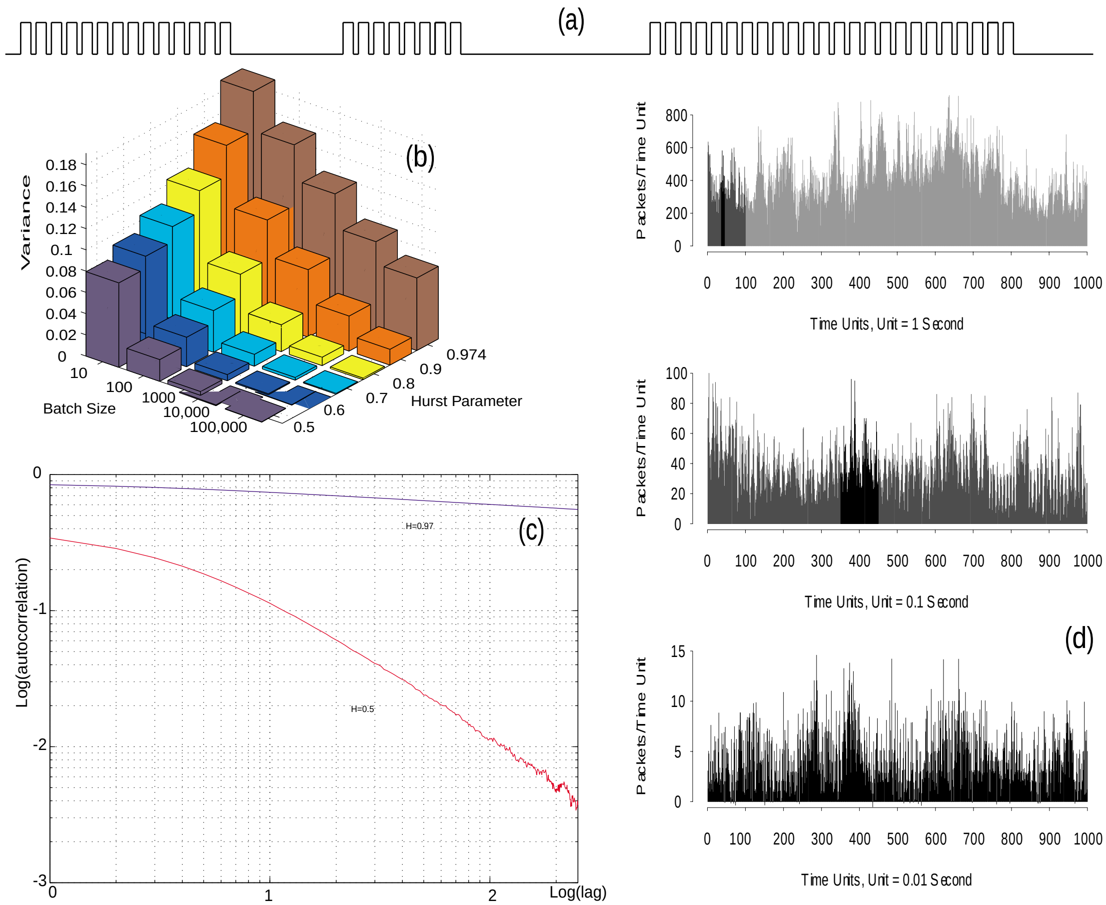

Figure 1. The difference between LRD and SRD traffic sources. (a) A bursty source of traffic. LRD traffic is characterised by long bursts in both the 'on' and 'off' states. (b) The effects of batch size on the variance of the average value as LRD is increased. At high H values variances remain relatively high as batch size increases: for low H values and SRD ($H = 0.5$) variances fall off rapidly with increasing batch size. (c) Decay in the autocorrelation function. The function decays rapidly for SRD sources ($H = 0.5$), but very slowly for sources with $H = 0.97$ (close to the maximum possible level of LRD). (d) Self-similarity in packet traffic. Plots of packet frequencies over different time scales reveal the same general appearance - that is 'self-similarity' (reproduced from Leland *et al*, [?])

used version of TCP.

Clearly the real internet is not a regular network! Much work has been done to describe its topology. It has been argued to be a scale-free network (see references given above) with a power law distribution of the probability of node connectivities(degrees) given by [?, ?] :

$$P(k) \sim Ck^{-\gamma} \quad (2)$$

with $\gamma \approx 2.22$. Here C is a constant and k is the node degree.

To model the actual internet more accurately we have used interactive growth (IG) networks [?]. These are a new type of scale-free network. They are generated by reflecting the process by which the internet grows in reality. The Interactive Growth (IG) model [?], resembles the dynamical properties that have been observed [?, ?] in the Internet history data. For example, newly added nodes have no more than two links connecting to already existing nodes; and new links not only connect new nodes to old nodes, but also connect between old nodes. The IG model matches well the degree distribution and the rich-club phenomenon of BGP and Traceroute as graphs, [?]. Zhou and Mondragon have shown in this paper that they accurately model both the node degree distribution and the tier structure. The internet exhibits a so-called *disassortive* mixing, where highly connected nodes are more likely pointing to less connected nodes.

The *rich-club* phenomenon emphasizes the fact that rich nodes are densely interconnected with each other. This property does not conflict with the disassortive mixing behaviour, because just a small number of connections between the rich nodes can make a significant difference on the network structure. The disassortive mixing is the reason that the rich-club phenomenon was ignored before, but it is an important detail of the complexity of the Internet topology.

As shown in Fig.2, there are actually two ingredients that can be responsible for accurately matching degree distribution and the rich-club phenomenon:

- New links not only connect new nodes to old nodes, but also connect between old nodes;
- Newly added nodes have no more than two links connecting to already existing nodes.

These two dynamical properties have been observed in detail, see [**?**, **?**], and also the Internet history data, where further references are available. However, whether a new link between old nodes starts from a host node (that a new node attaches to), or starts from a chosen old node, has not yet been supported by analysis on the actual measurement data. The reason that the Bu, [**?**] or Dorogovtsev, [**?**] models do not represent the Internet as accurately as the IG model could be because all their new nodes are connected by the same number of new links (for the Internet, it is set to be 3).

New nodes, equivalent to new customers, bring new traffic load to rich nodes which act as service providers would in real networks. This results in both an increase in traffic volume and a change in the traffic pattern around the rich nodes. This change in turn triggers the addition of new links between the rich nodes and other nodes in order to balance network traffic and optimise network performance.

In our previous simulations with regular networks we used the same service rate for each node. However, the node connectivities are all equal in a regular network. By contrast, some nodes in IG networks have very high degrees, and using the same service rate for each node would be inappropriate. Hence we relate the service rate at each node to its connectivity.

2 Network Traffic Model

We based our simulations on an IG network. The exponent γ in eqn. (2) was fixed as 2.22 as is claimed for the real internet, [**?**]. Each node can be either a host or a router. Routers contain a single routing queue that receives packets in transit across the network, and releases them back into the network. The *service rate*, s, at which packets are released by routers is determined by the equation:

$$s = 0.25n^a \tag{3}$$

The simulation is of the fixed increment time advance type [**?**], so that the rate s may be measured in packets per time tick of the simulation. In eqn. (**??**): n is the node degree, and a represents the server strength. The parameter a ranges between 1 and 2 in the results presented here.

Hosts have the same routing function as routers with similar routing queues, but they also act as sources and each has a transmit buffer. LRD and Poisson traffic sources create packets that are stored in the transmit buffers until they may be released into the network. The way in which this is done is described below.

We use a simplified version of TCP Reno [**?**] as the network transfer mechanism. Reno is the most widely used version of TCP at present. Our version originates from that described by [**?**]. The two main mechanisms used by TCP Reno are called *slow-start* and *congestion avoidance*. Slow-start is described in detail below. It is the mechanism applied when files start, or packets sent have not been acknowledged within a certain time limit. Congestion avoidance is applied when the slow-start mechanism has reached its limit. It only affects long files in networks working well within their capacity. Our simplified version only applies to slow-start because in the actual internet this mechanism affects all connections and also dominates for most connections. Congestion avoidance has a much smaller effect by comparison.

We use the family of *Erramilli* interval maps as the basis for each LRD traffic source, [**?**], $f = f_{(m_1,m_2,d)} : I \to I$, where

$$f(x) = \begin{cases} x + (1-d)\,(x/d)^{m_1}\,, & x \in [0,d], \\ x - d\,((1-x)/(1-d))^{m_2}\,, & x \in (d,1], \end{cases} \tag{4}$$

where $d \in (0,1)$. The parameters $m_1, m_2 \in [3/2, 2]$ induce *map intermittency*. The map f is used to produce an *orbit* of real numbers $x_n \in [0,1]$. This orbit is converted into a binary *on-off* sequence where *'Off'* is represented by values falling in the interval $[0,d]$, and *'On'* by values falling in the interval $(d,1]$. It should be noted that all possible binary sequences are available from the map f. If the map f is in the *'On'* state, each iteration of f represents a packet generated. One sojourn period in the *'On'* side of the map represents a whole file. These *files* are then *windowed* using the slow-start algorithm, adding another dynamical layer to the system. The algorithm is as follows:

At a given host i in the network, and time $t = n$, there is a current state, $x_i(n)$, and a current window size, $w_i(n)$, for the number of packets that can be sent at time $t = n$. The window size has a maximum value w_{max}. There is also a residual file size, $s_i(n)$, at node i which is given by the number of iterates of f such that $f^{s_i(n)}(x_i(n)) > d$, and $f^{s_i(n)+1}(x_i(n)) < d$. The source will send $p_i(n) = \min\{w_i(n), s_i(n)\}$ packets. The full dynamics therefore takes the form, (see Erramilli *et al.*, 2002, [**?**]) :

1. $x_i(n) < d$, i.e. no packet generated -

$$w_i(n+1) = 0, \quad \text{and} \quad x_i(n+1) = f(x_i(n)); \tag{5}$$

2. $x_i(n) > d$, i.e. packet generated -

$$w_i(n+1) = \begin{cases} 1, & \text{if} \quad x_i(n-1) < d, \\ \min\{2w_i(n), w_{max}\}, & \text{otherwise}, \end{cases} \tag{6}$$

and $x_i(n+1) = f^{p_i(n)}(x_i(n))$.

This algorithm applies if all packets in a window are acknowledged before the *retransmission timeout* (RTO) limit is reached. If packets take longer than this to be acknowledged the window is sent again with the RTO doubled and the window size set to zero. When the map is *'Off'* the window size is zero and no packets are sent.

This initial value of RTO is calculated using the exponential averaging method, [**?**]. This method keeps a running average of all round trip times. This average is weighted towards more recent round trips, and is used in calculating the RTO.

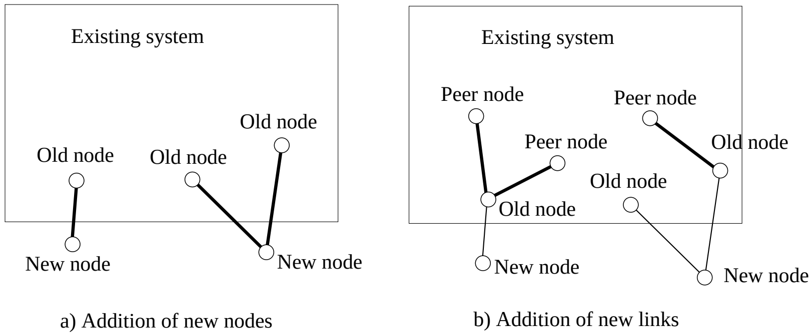

Figure 2. The evolution of an interactive growth(IG) network.

Our routing algorithm uses a pre-calculated look-up table of shortest paths. All links between nodes are assumed to have unit length. At each time step packets are forwarded from the head of each routing queue. If an acknowledgement packet reaches its destination, this triggers the release of the next window of packets from that host. The number of packets forwarded from the routing queues is given by eqn. (3).

A simple set of rules are used for forwarding packets from the routing queue or *source*:

- The shortest path to the destination for each node neighbouring the source is read from the look-up table. A neighbour that is the minimum distance from the destination is selected.
- If several neighbours share this minimum distance, the neighbour that has had the fewest packets forwarded to it by the source is chosen.
- If the number of packets forwarded are equal a random selection is made.
- If the neighbour chosen is at its routing queue length limit the packet is dropped.

3 Simulation Results

This paper presents an extension of our previous work with regular lattices and open-loop traffic, see [**?**, **?**].

In Fig. 3, a comparison is made between the different topologies and packet transport algorithms used in this and the previous work. Four combinations are considered:

- an IG network with a TCP closed-loop algorithm;
- a Manhattan network with an open-loop algorithm;
- an IG network with an open-loop algorithm; and
- a Manhattan network with a TCP closed-loop algorithm.

The network size is 1024 nodes and the host density is 589/1024 in all cases. The server strength index a of eqn. (2) is set to 1 for all four combinations. For the Manhattan network, hosts are selected randomly; for the IG network, connectivity 1 and 2 nodes are selected to be hosts. This reflects the real case more closely than a random distribution. (The difference that this makes is analysed in Fig. 7 below).

In Fig. 3(a) average lifetimes are plotted against load for the four cases with LRD traffic sources at each host. Here the average lifetime is simply the average time spent by packets in the network. The load is the average number of packets produced by each traffic source per time tick of the simulation.

Note that average lifetimes include the waiting time in transmit buffers. Measurements from actual networks would not include this. In addition loads quoted are those offered to the transmit buffers, rather than the loads exiting from the buffers. We have arranged things this way to allow the end users experience to be modelled.

Clearly the closed-loop TCP algorithm results in much longer lifetimes for both types of network. The reason for this becomes clear when you consider TCP. The requirement that packets be acknowledged before the next window is sent is quite conservative. Hence new windows cannot be sent by hosts more frequently than the round trip times. Congestion is responded to immediately because it causes an increase in RTT's and makes sources back off. Since file sizes are not so large, and window sizes are often reset to 1 due to the RTO limit, throughput can never be that high. This results in packets being delayed in the transmit buffer; this is the primary cause of increased packet lifetimes. In contrast the open-loop algorithm does not react to congestion and allows queues to build up at routers. At high enough loads the open-loop networks become congested and lifetimes approach those of the closed-loop networks.

In addition, Fig. 3(a) shows that the network topology is also a very important factor. In fact, a Manhattan network with an open-loop algorithm has longer lifetimes than an IG network with a closed-loop algorithm. This is because of the much shorter average path lengths in the IG network. The average path length in the Manhattan network is 16 'hops' (one 'hop' is the distance between neighbouring nodes). In an IG network ordinary nodes connect to rich nodes with a high probability, and rich nodes also connect to one another preferentially. This leads to much shorter average path lengths. For the IG network used here the average path length is only 5 hops.

In Fig. 3(b) the same measurements are made with

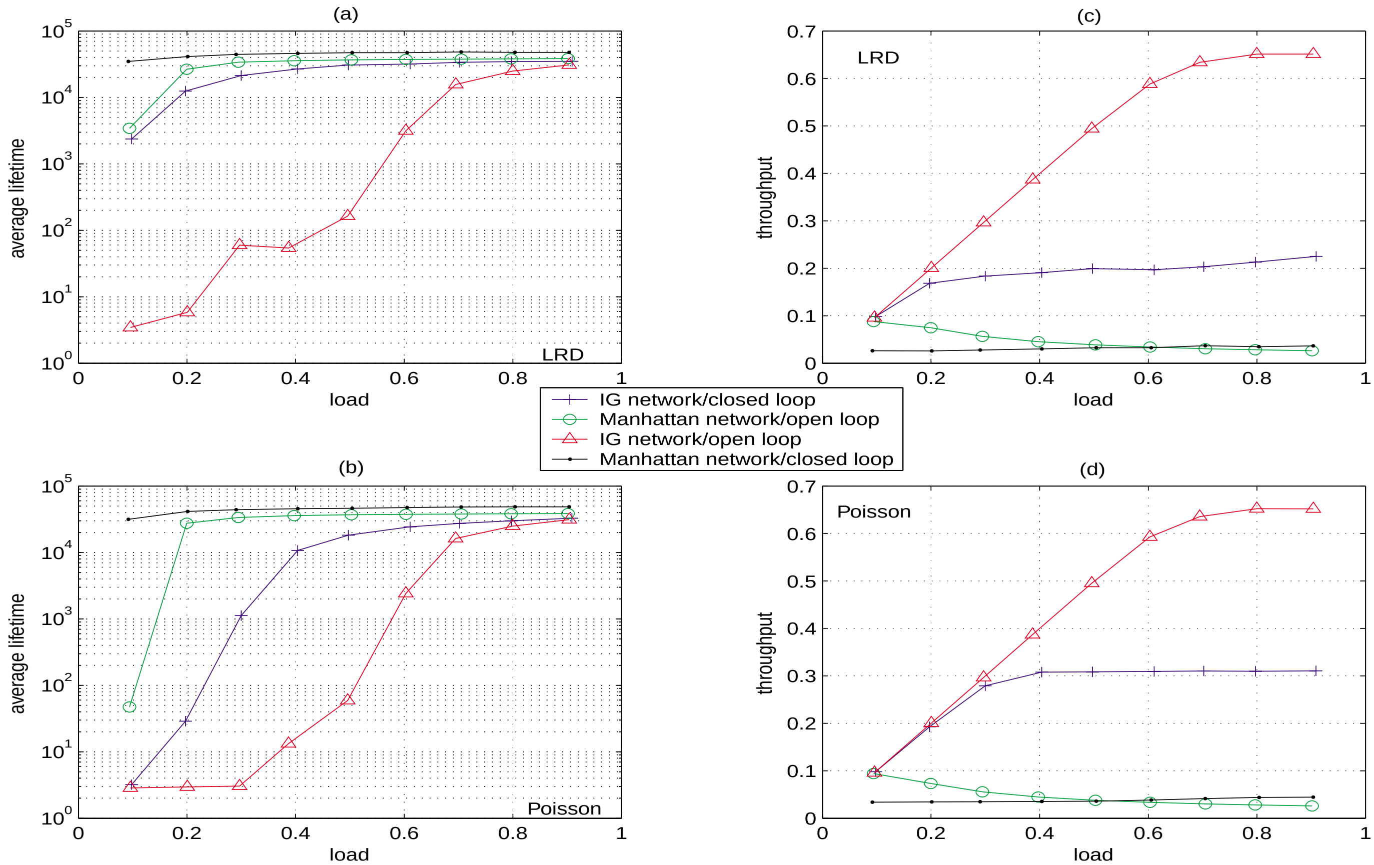

Figure 3. Comparisons between the open-loop simulations and closed-loop simulations on regular networks and scale-free IG networks. Network sizes and host densities are the same for all simulations.

Poisson traffic sources substituted for LRD sources. Results are very similar. The earlier onset on congestion reported in [**?**, **?**] is still present.

Fig. 3(c) shows throughput plotted against load. The throughput is defined as the number of packets reaching their destination per unit time per host. Results are consistent with those for average lifetime. Identical networks using open-loop algorithms have higher throughputs; the IG network performs more efficiently for both types of algorithm. Similar behaviour is observed for Poisson sources.

As outlined above, regular open-loop simulations are fundamentally different to the closed-loop simulations on IG networks used in this paper. For this reason more detailed comparisons between these simulations and our earlier work is not possible.

Fig. 4 shows a comparison of different server strengths. The two types of source, SRD and LRD, are again considered. The same IG network with the same pattern of hosts is used. Results from LRD sources and Poisson sources differ greatly. Throughputs (Figs 4(c) and 4(d)) at the lower server strengths (a of equation (3) is set equal to 1 and 1.1) are qualitatively similar: the throughput matches the load up to a threshold and then levels out. However, this threshold is more than 50% higher for the Poisson sources. If a is increased to a value of 1.5 this threshold can no longer be seen; throughputs for the two source types are similar. When servers at 'rich' nodes are strongest (i.e. $a = 2.0$) the situation is reversed: the network with LRD sources has a higher throughput, able to handle the maximum load applied to it without becoming overloaded.

Figs 4(a) and 4(b) show average lifetimes for the two source types. At low loads behaviour is similar to that seen in our earlier work with regular lattices and open-loop protocols: lifetimes for Poisson sources are much less than for LRD. In the case of Poisson sources there is a similar transition from the free state to the congested state. At increasing server strengths the transition becomes less pronounced. This is due to the increased network capacity which slows the onset of congestion.

In Fig. 5, 3D plots show transmit and routing queue lengths for the same network. Again both types of SRD and LRD traffic sources were used. Nodes are arranged in ascending order of connectivity. In Fig. 5(a) and 5(b) transmit queue lengths are linked to the average lifetimes for both types of source. At low loads they are very short; as loads increase they rapidly rise in length.

The routing queues (Figs 5(c) and 5(d)) are much shorter. There is no pattern to the queue lengths for LRD sources, with the exception that all queues arise at the higher connectivity nodes. Poisson sources produce different behaviour.

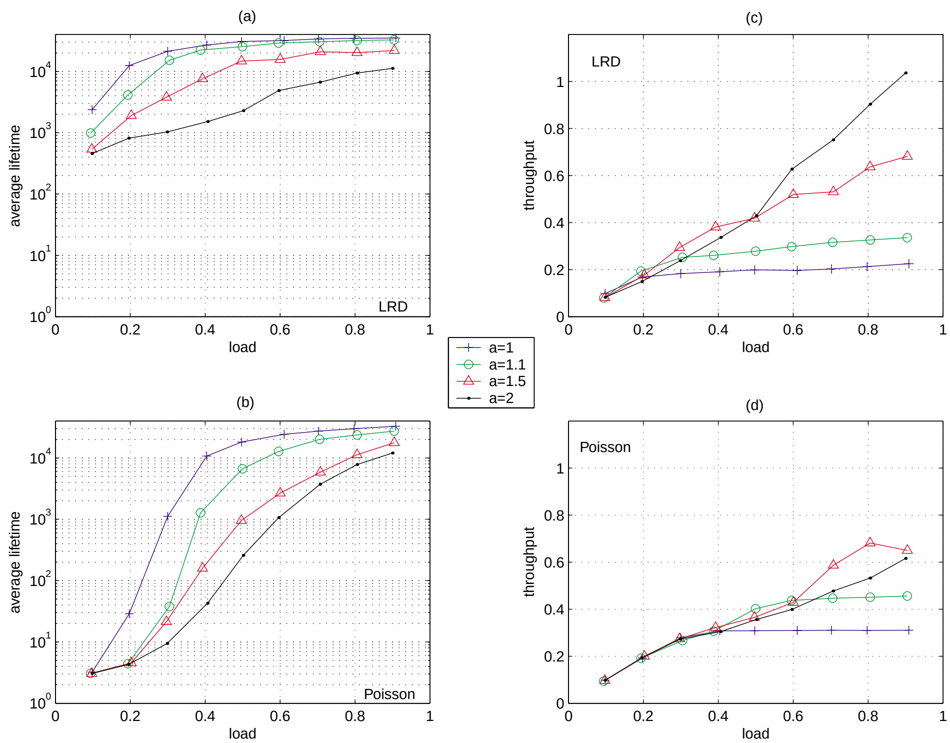

Figure 4. The number of packets that can be served at each time instant is increased according to a power law n^a, where n is the degree of the node and $a = 1, 1.1, 1.5, 2$.

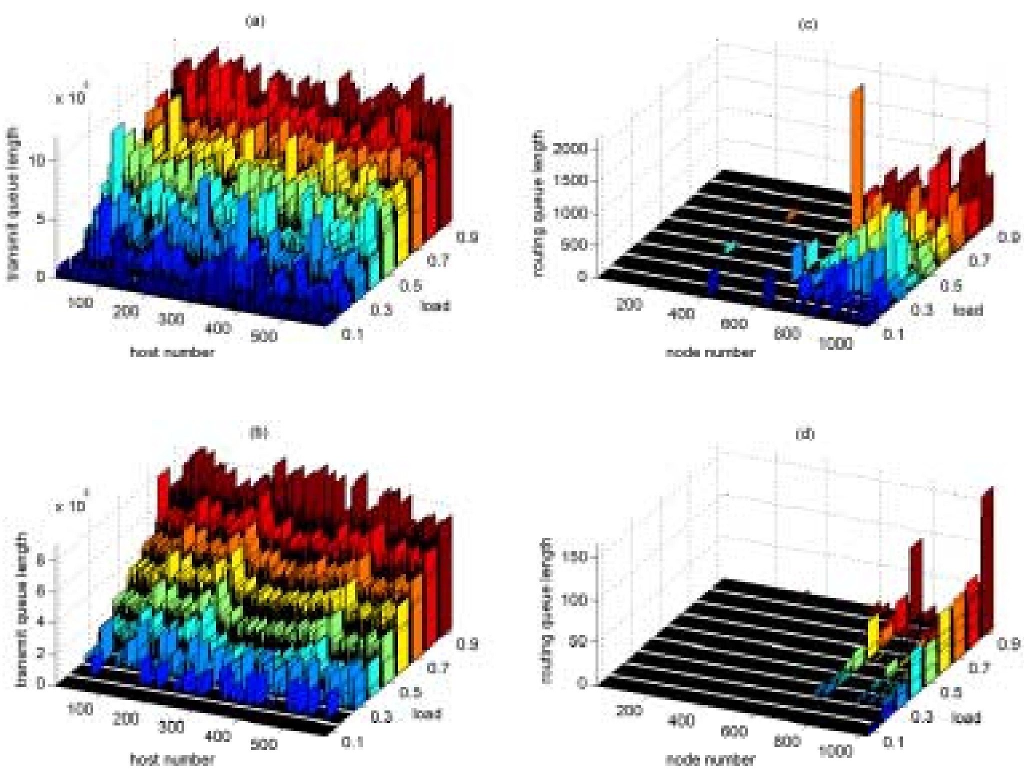

Figure 5. Queue lengths for host nodes of an IG network are shown as load increases, for both Poisson and LRD sources.

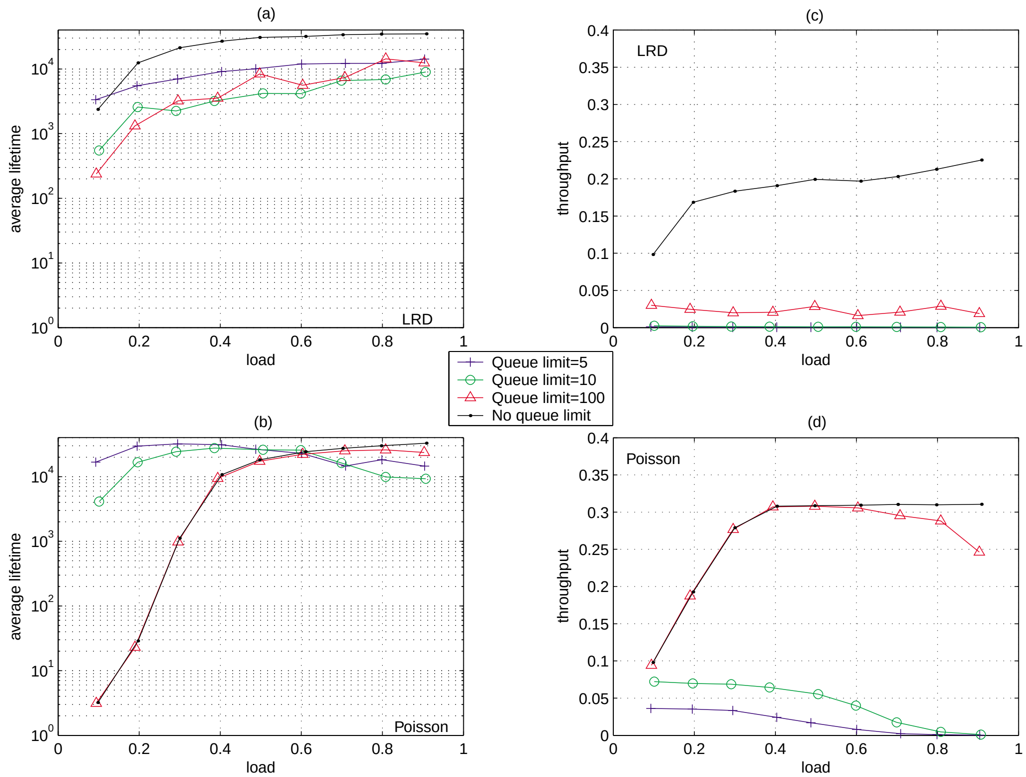

Figure 6. Effect of varying queue length limit on average packet lifetime and throughput for LRD and Poisson sources.

Queue lengths are much less and queue lengths can be seen to increase with increasing load. In addition the same nodes have queues at different loads. This implies that host distribution is important. The highest connectivity nodes build up queues for all loads. As might be expected these nodes are the most congested. The server strength, a, is only 1 here. At higher server strengths connectivity becomes less significant as would be expected.

The fact that much longer queues form in the transmit buffers shows that these queues provide the main contribution to average lifetimes.

In Fig. 6 we have simulated the packet dropping of real networks by limiting the routing queue lengths. The same network and host pattern as described for Fig. 4 has been used with a server stength, a, of 1. Very severe packet loss has been modelled here in order to test the extreme situation. Real networks generally suffer much less packet loss.

For LRD sources (Fig. 6(a)) average lifetimes are greatly reduced when the queue limit is decreased. When queue lengths are limited packets are dropped at the routers and therefore re-sent more frequently. This causes shorter waits in the transmit buffer. This can be seen most clearly at the very low queue length limit.

In the case of Poisson sources (Fig. 6(b)) average lifetimes behave quite differently. Lifetimes peak at a load of 0.3 for the queue limit of 5. This peak shifts to higher values as the queue limit increases. When there is no queue limit the lifetime has the 's' shape seen previously in regular networks.

Fig. 6(c) shows throughputs for LRD sources. These are greatly reduced when a queue limit is applied. At the smaller queue limits throughput is close to zero. In the case of Poisson sources (Fig. 6(d)) throughput is similar for the queue limit of 100, but also much less at the lower queue limits. However, throughputs are still much higher than for the LRD sources. This is caused by the shorter queues in the case of Poisson sources. Most queue lengths are less than a 100, meaning that this limit has little effect.

In Fig. 7 the same IG network has been used, but hosts have now been selected randomly with the same density of 589/1024. Results are similar. The network with randomly placed hosts always performs slightly better than the one with hosts placed at the low degree hosts. Average lifetimes are lower and throughput is slightly higher. This shows that the inclusion of a small number of rich nodes in fact makes little difference.

All our measurements have been repeated with different host patterns and IG networks, but keeping host density and network topology parameters the same. Results have been very similar indicating a good degree of robustness in our data.

4 Conclusion

In the work presented here we have carried on from our previous simulations of regular lattices with open-loop packet

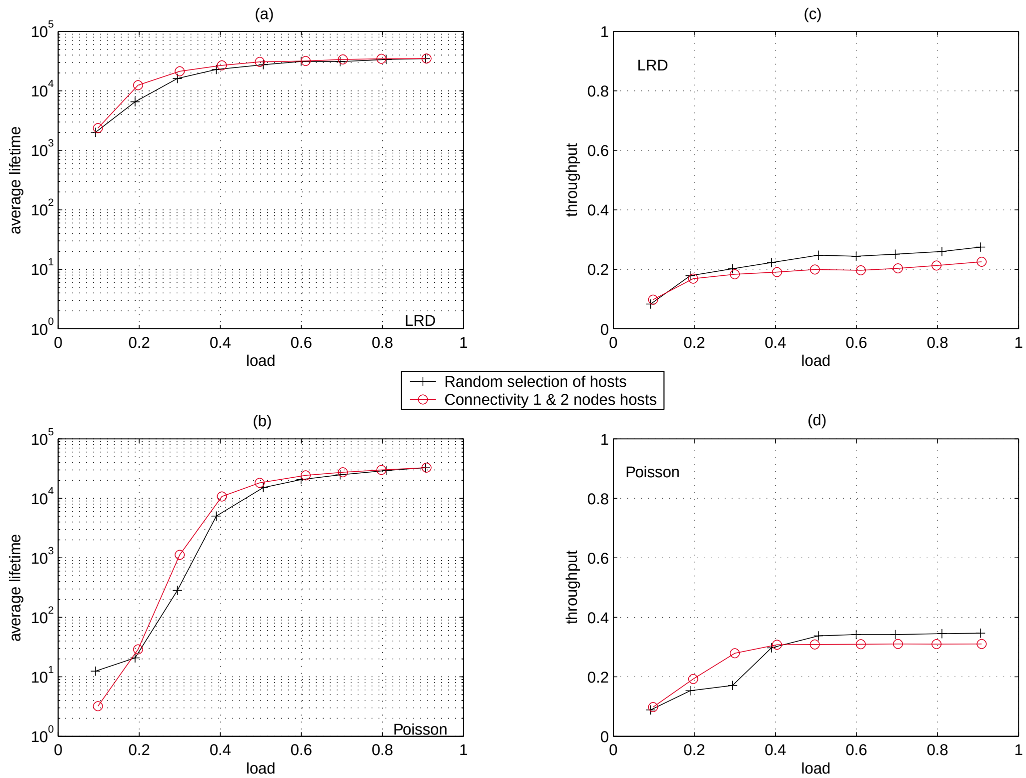

Figure 7. Average lifetime and throughput characteristics for different topological spreads of hosts in an IG network.

transport mechanisms. By using large IG networks and TCP our simulations reflect the real internet more closely.

IG networks and other scale-free networks clearly have very different properties to regular networks. In scale-free networks a small number of nodes have very high degree and path lengths are much shorter. TCP is also a closed-loop rather than an open-loop transport mechanism. The two are quite different in nature. These differences mean that only a qualitative comparison is possible between this and our previous work.

Measurements are robust in that changes in network and host pattern produce similar results. This fact endorses the IG method as a means of characterising real scale-free networks.

As in previous work we have compared LRD and Poisson (SRD) sources. As before the two source types produce results that differ greatly. An important phenomenon is that LRD sources can cause severe local congestion when several highly connected nodes are adjacent. This congestion can be greatly reduced by using higher server strengths. For very high server strengths, networks with LRD sources may even perform better than those with Poisson sources.

Our results show that severe packet loss can have a dramatic effect on throughput. However, to improve the utility of our results more consideration needs to be taken of the mechanisms and levels of packet loss in the real network.

We have seen what the user experiences when the network is overloaded. If the load were very low, TCP would prevent congestion ever occurring. TCP controls congestion within the network quite effectively, but the price is a greatly reduced throughput. This is seen by the user as time spent waiting for connections and increased download times.

In future work we intend to model larger networks and also consider networks that have dynamical topologies.

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References

[1] R. Albert and A-L. Barábasi, Statistical Mechanics of Complex Networks, *Rev. Mod. Phys.* **74**, **47** (2002).

[2] D.K. Arrowsmith, R.J. Mondragon, J.M. Pitts, M. Woolf, Internet packet traffic congestion, *Proc Int. Sym. Cir. and Sys.*, 2003, ISCAS '03., **3**, 2003, 746 -749.

[3] T. Bu and D. Towsley, On Distinguishing between Internet power law topology generators. *Proc. of INFOCOM* 2002.

[4] Q. Chen, H. Chang, R. Govindan, S. Jamin, S. J. Shenker, and W.Willinger, "The Origin of Power Laws

in Internet Topologies (Revisited)," Proc. of INFOCOM 2002, 2002.

[5] S.N. Dorogovtsev and J.F.F. Mendes, *arXiv:cond-mat/0009065*, 2000.

[6] A. Erramilli, R.P. Singh and P. Pruthi, *Proc. of the 14th Int. Teletraffic Conf.*1994, edited by James W. Roberts (Elsevier, Amsterdam, 1994), 329-38.

[7] A. Erramilli, M. Roughan, D. Veitch and W. Willinger, Self-Similar Traffic and Network Dynamics, *Proc. IEEE*, **90**, 2002, 800-819.

[8] M. Faloutsos, P. Faloutsos, and C. Faloutsos, On Power-Law Relationships of the Internet Topology, *Proc. ACM/SIGCOMM, Comput. Commun. Rev.*, **29**, 251-262, 1999.

[9] H.E. Hurst, Methods of using long-term storage in reservoirs, *Proceedings of the Institution of Civil Engineers*, Part I, pages 519–577, 1955.

[10] A.M. Law and W.D. Kelton, Simulation Modeling and Analysis, 3rd Edition, *McGraw-Hill*, 2000.

[11] W.E. Leland, M.S. Taqqu, W. Willinger and D.V. Wilson, On the Self-Similar Nature of Ethernet Traffic. *Proc. ACM SIGCOMM* 93, 1993.

[12] The Network Simulator 2, ns2 - http://www.isi.edu/nsnam/ns/.

[13] Opnet, Opnet Tech. Inc.-http://www.opnet.com.

[14] W. Stallings, Data and Communications,*Prentice Hall Int. Inc.*,2000.

[15] W. R. Stevens, TCP/IP Illustrated, Volume 1: The Protocols, *Addison-Wesley*, 1994.

[16] L. Subramanian, S. Agarwal, J. Rexford and R. H. Katz, Characterizing the Internet Hierarchy from Multiple Vantage Points, *Proc. of INFOCOM 2002*, June, 2002.

[17] A. Vázquez, R. Pastor–Satorras and A. Vespignani, "Large–scale topological and dynamical properties of the Internet," Physical Review E, vol.65-066130, 2002.

[18] X.F. Wang, Complex Networks: Topology, Dynamics and Synchronization, *International Journal of Bifurcation and Chaos*, **12**, 885-916, 2002.

[19] M. Woolf, D.K. Arrowsmith, R.J. Mondragon, J.M. Pitts, Optimization and phase transitions in a chaotic model of data traffic, *Phys. Rev E*, 66, 046106 (2002).

[20] S. Zhou and R. J. Mondragón, 'Towards modelling the Internet topology – the Interactive Growth model,' Proceedings of the 18th International Teletraffic Congress, Berlin, Germany, 2003 (Elsevier Science, Amsterdam, 2003)(cs.NI/0308029), Sept. 2003 , p. 121.

[21] S. Zhou and R. J. Mondragon, The rich-club phenomenon in the Internet topology, *IEEE Comm. Lett.* , 2004, **8**(3), 180-182.

[22] S. Zhou and R. J. Mondragón,Redundancy and Robustness of the AS-level Internet Topology and its Models *IEE Elec. Lett.* **40**(2), 2004, 151-152.