MANMADE

DIAGNOSING VULNERABILITY, EMERGENT PHENOMENA, and VOLATILITY in MANMADE NETWORKS

Modelling critical network infrastructures

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Collaborators

- Collegium Budapest
- European Commission, JRC(Ispra)
- Macedonian Academy of Sciences and Arts
- Queen Mary University of London
- Università Carlo Cattaneo
- Stakeholders
  - National Emergency Supply Agency, FINGRID (Finland)
  - UK-Electricité de France

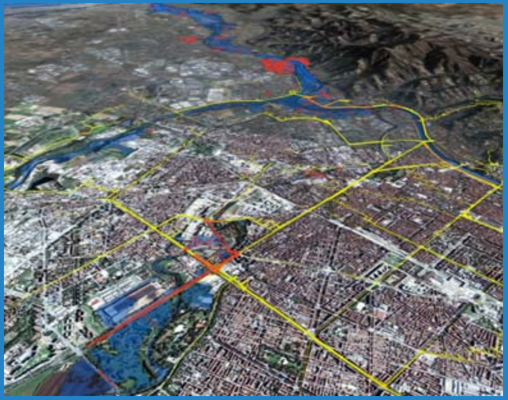
NETWORKS

- Social
- Energy
- Transport
- Communication
- ……compound networks

What are the correct questions to ask and the most useful analytical tools to handle them?

MANMADE – its scope

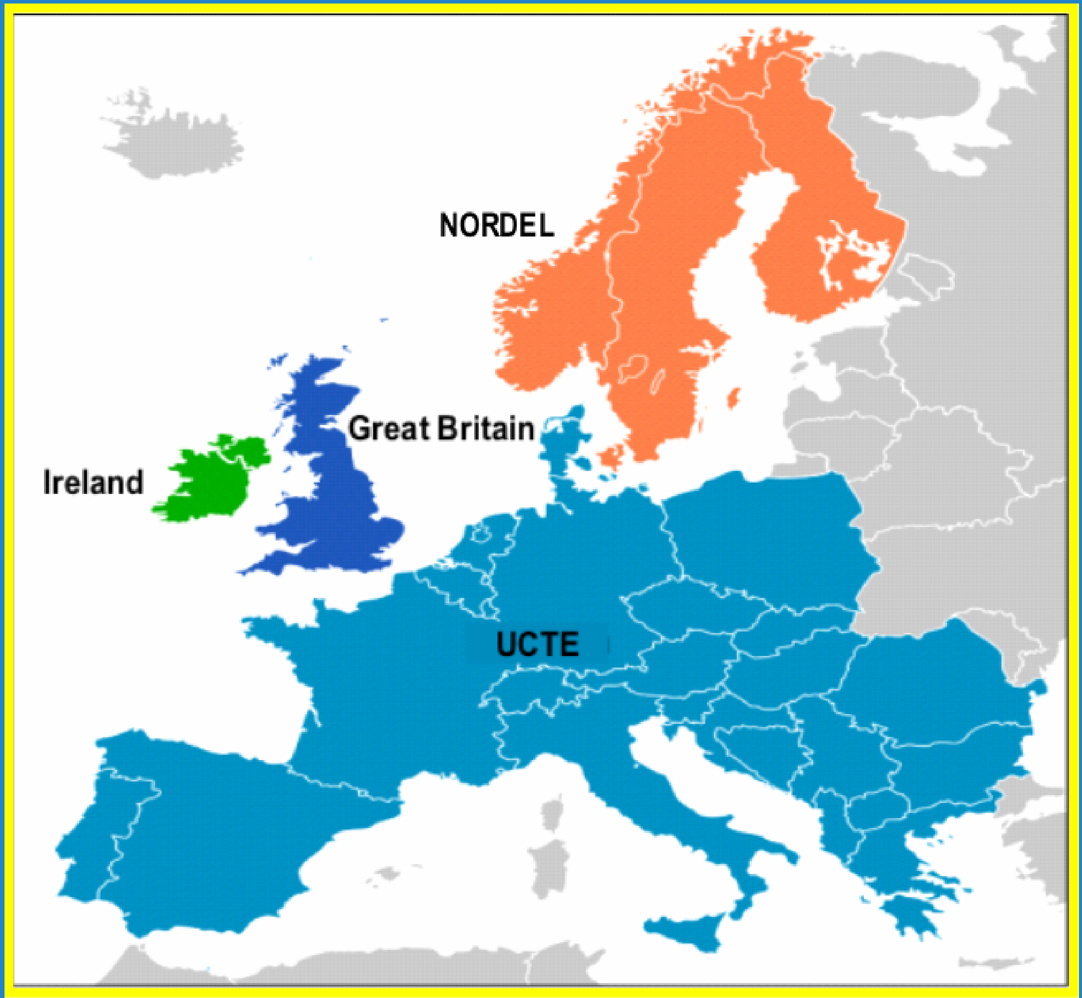
- The project concerns the compound networks that comprise Europe's critical infrastructure -
  - primarily on energy supply, emergency response systems and subsidiary key infrastructures
- Aim -
  - is to assemble, develop and apply mathematical methods to analyse large, man-made multi-element infrastructure systems



First steps - datasets

Data sets of major gas lines and exchange flows	Data sets of major gas lines between and into Western Europe
Datasets of spot price electricity	NORDPOOL time series spot price electricity in European markets
Spatial and topological maps of the road network	Urban street network of Milan, Turin and London
High voltage electricity grid	European Electricity Lines by synchronous regions European energy interconnected network (electricity and gas)

Electricity regions



Four independent synchronous electricity regions to be investigated

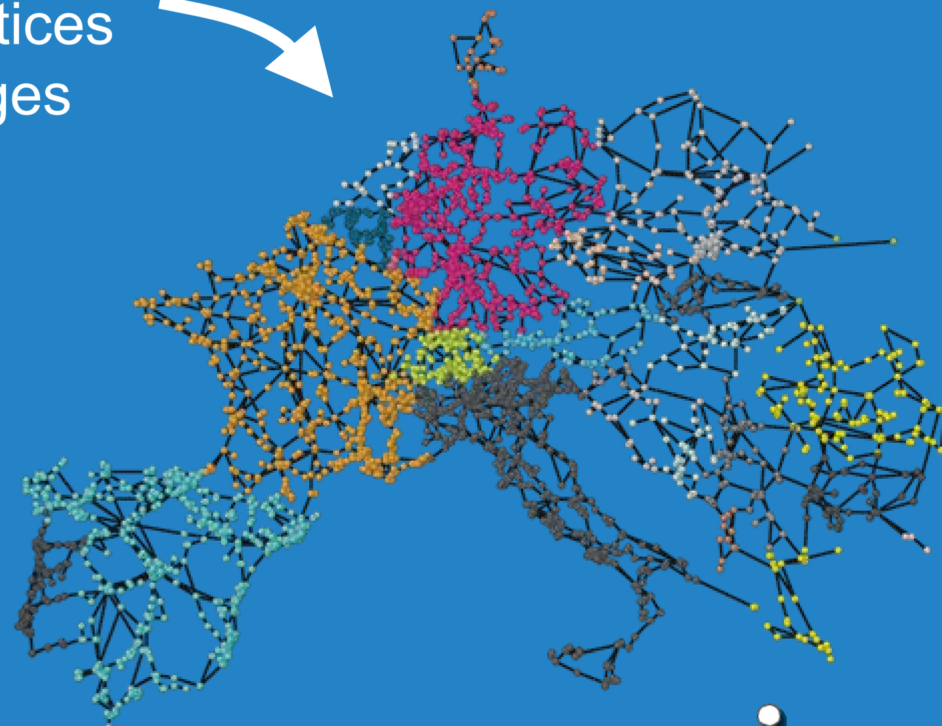
Network

- Scaling
- Topology
- Weighting

Electricity networks

NORDEL:
526 vertices
638 edges

UCTE:
4200 vertices
5305 edges



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Vulnerability

The more critical the component the more severe is the damage to the system when it is lost

Criticality of the component ⟷ Vulnerability of the system

Critical component = WEAK + IMPORTANT

WEAK: How susceptible to malfunction?

IMPORTANT: The consequences are great

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How to tackle this issue?

Characterize the measures of importance

Analyzing the network

Use these quantities as selection criteria for removal of the components

Attacking the network

Measures for the consequences

Results

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Network graphs and incidence matrix

- Graph G consists of a set of N nodes $V=\{v_1, v_2, v_3, \dots, v_N\}$ and a set of M edges $E=\{e_1, e_2, e_3, \dots e_M\}$
- An edge connects two nodes $e=(v_i, v_j)$ – there can be multi-edges
- The *incidence* between the edges and the nodes is recorded via an *adjacency matrix* $A=(a_{ij})$
- If edge j is joined to node i, then $a_{ij}=1$, otherwise 0
- The *degree* of node i is the # of `1' 's in row i of A
- The indicators $a_{ij}=1$ or 0 can be changed to *weights* w_{ij} where if $a_{ij}=0$, then $w_{ij}=0$
- The *Laplacian matrix* $L=(l_{ij})$ is an N x N matrix with

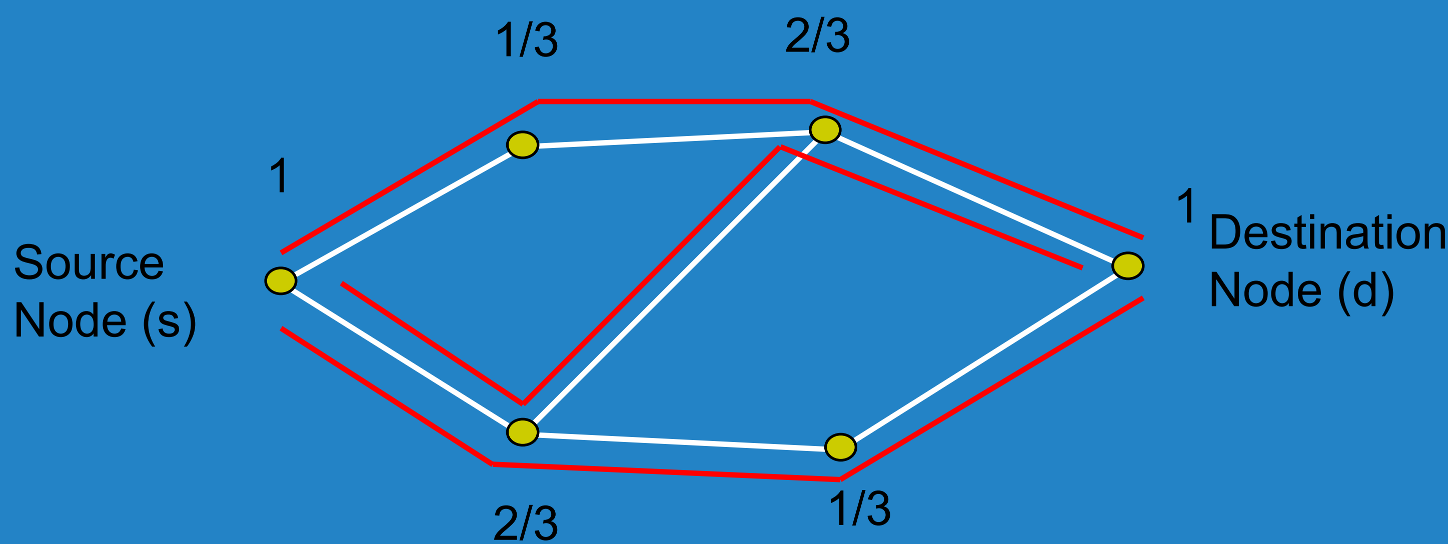
$L_{ij}=\deg(i)$ for $i=j$, $L_{ij}= -1$ for $i\neq j$ and v_i is adjacent by an edge to v_j, otherwise 0.

Relevance of the Laplacian matrix

- The *algebraic connectivity* of a graph *G* is the second-smallest eigenvalue of the Laplacian Matrix of *G*
- This eigenvalue is greater than 0 if and only if *G* is a connected graph
- This is a corollary to the fact that the number of times 0 appears as an eigenvalue in the Laplacian is the number of connected components in the graph
- The magnitude of this value reflects how well connected the overall graph is, and has implications for properties such as synchronizability and clustering

Betweenness centrality

- *Betweenness centrality* measures the importance of nodes in terms of the frequency of their appearance on shortest paths



- *Betweenness* of node i for the pair *sd* = relative number of shortest paths between s and d which visit node I
- *Betweenness* of node i for graph requires calculation for all pairs of vertices

Analysis of weighted undirected networks

Graph -> Network ▶ $G = (V, E)$

Vertices (substations)

Edges (electrical lines)

Adjacency matrix $A(G)$

defines which vertices are connected

$$a_{ij} = \begin{cases} 1, \text{if } v_i \text{ and } v_j \text{ are neighbours,} \\ 0, \text{otherwise} \end{cases}$$

Weights matrix $W(G)$

defines the weights on the edges

$$w_{ij} = \begin{cases} w_{ij}, \text{if } a_{ij} = 1, \\ 0, \text{ if } a_{ij} = 0 \end{cases}$$

Weights = Capacity of the electrical line (kV)

[220kV – 2500kV]

Measures of importance

Inverse "distance" from the given node to every other node in network - efficiency

$$C_i = \sum_j \frac{1}{d_{ij}}$$

shortest paths in the graphs

$$d_{ij}$$

$$B_i = \sum_v \sum_u \frac{n_{uv}(i)}{n_{uv}}$$

The proportion of all shortest paths in the network that run through a given node

Degree

Efficiency / Closeness

Betweenness

Centrality indices

Modal index

$$D_i = \sum_j a_{ij}$$

or

$$D_i = \sum_j w_{ij}$$

Spectral decomposition of the Laplacian matrix

$$L = D - W \rightarrow \lambda_i \phi_i = L\phi_i$$

$$\Gamma = L'\Phi \rightarrow M_i = \sum_j |\gamma_{ij}|$$

Sum of modal contributions (load in each mode) for each vertex

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Weighted vs. unweighted node ranking

of Nordel network

Spearman ranking correlation coefficients for Efficiency, Betweenness and Spectral analysis

NORDEL	Eff-W	Eff-UnW	BC-W	BC-UnW	SA-W	SA-UnW
Eff-W	1.00	0.89	0.34	0.32	0.22	0.58
Eff-UnW	0.89	1.00	0.42	0.44	0.55	0.72
BC-W	0.34	0.42	1.00	0.96	0.51	0.61
BC-UnW	0.32	0.44	0.96	1.00	0.57	0.62
SA-W	0.22	0.55	0.51	0.57	1.00	0.76
SA-UnW	0.58	0.72	0.61	0.62	0.76	1.00

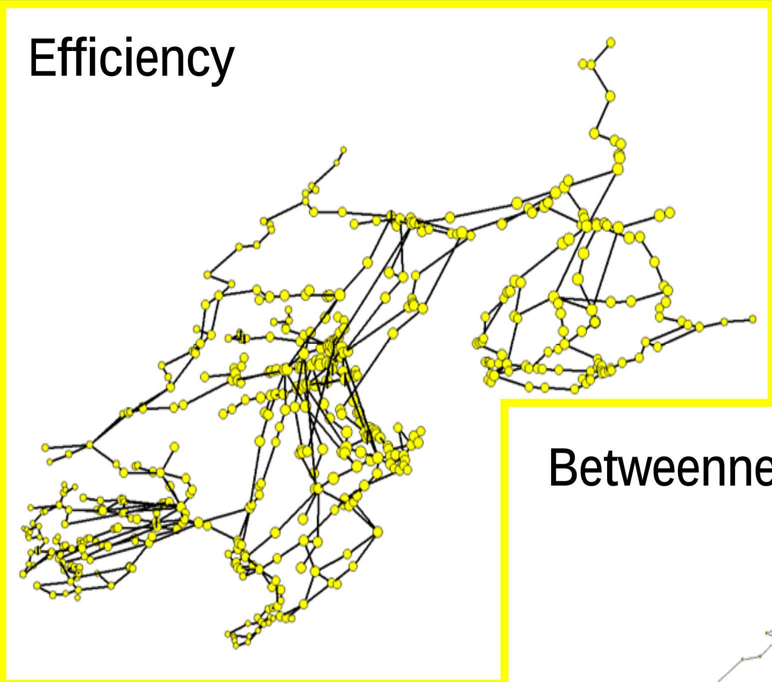
WEIGHTED ⟷ UNWEIGHTED

BETWEENNESS ⟷ EFFICIENCY

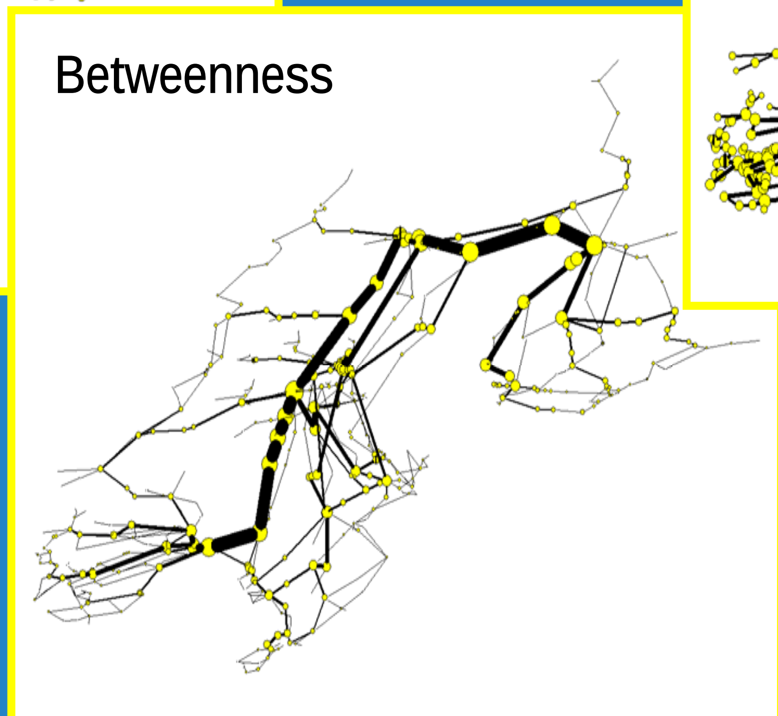
SPECTRAL ANALYSIS

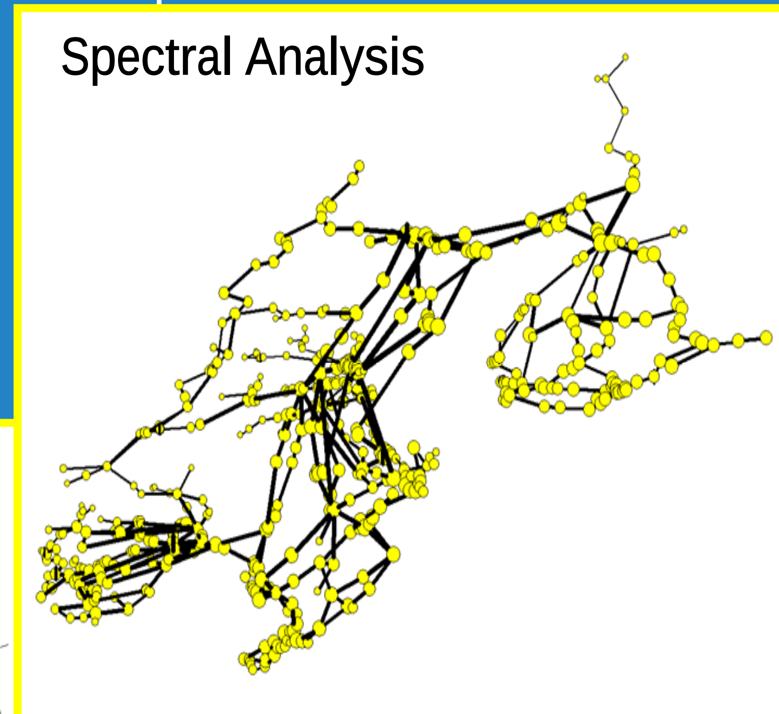
Ranking

Size of the vertices
Width of the edges → the relative value of the importance



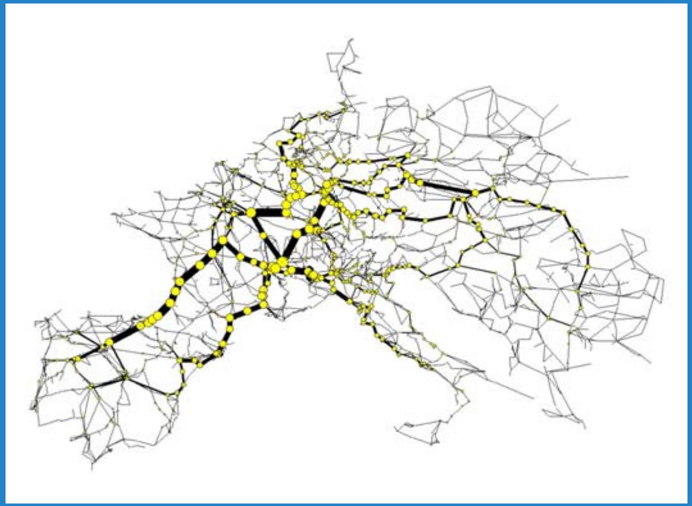
NORDEL





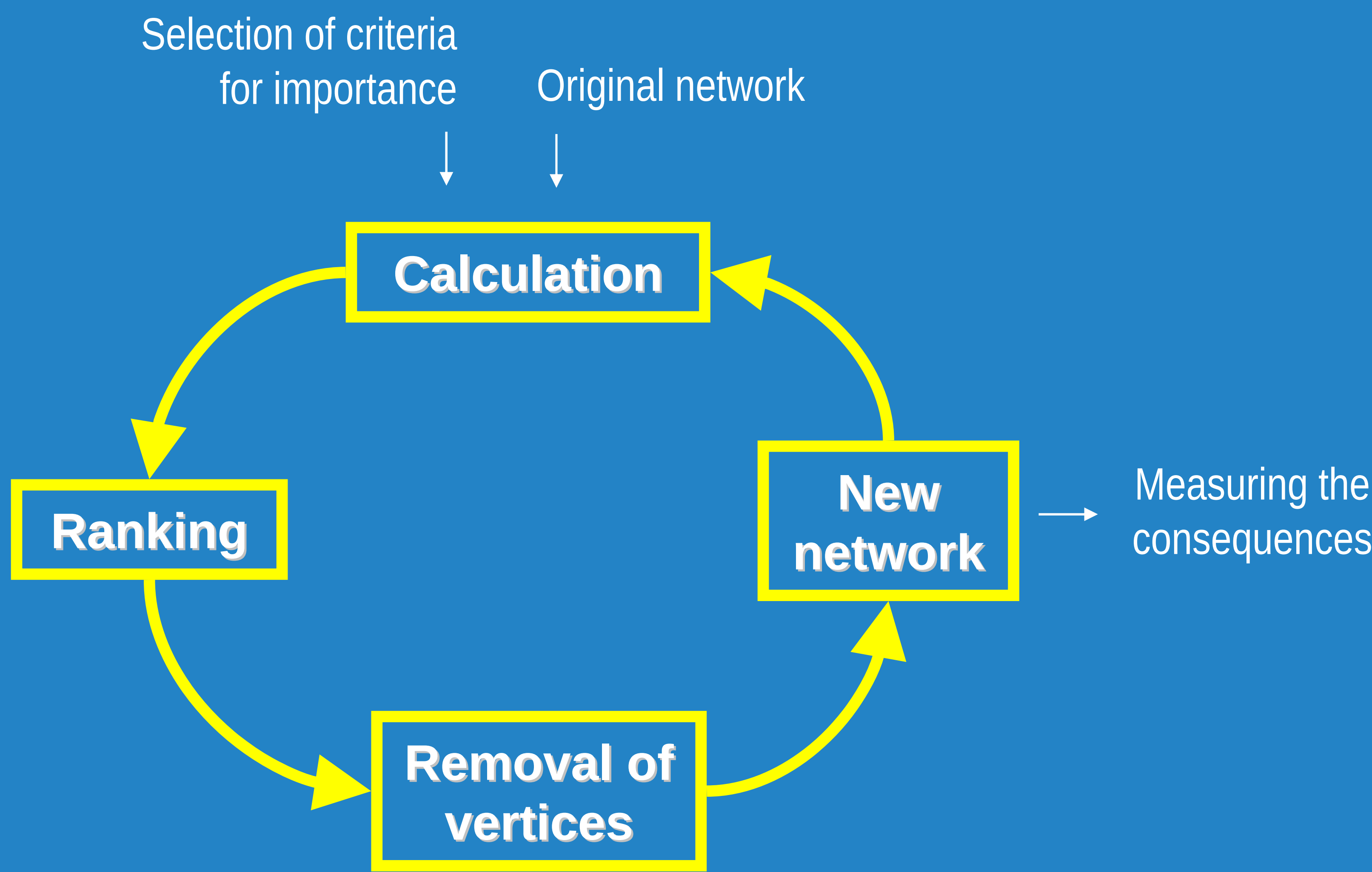
Efficiency –choice of metrics

NORDEL

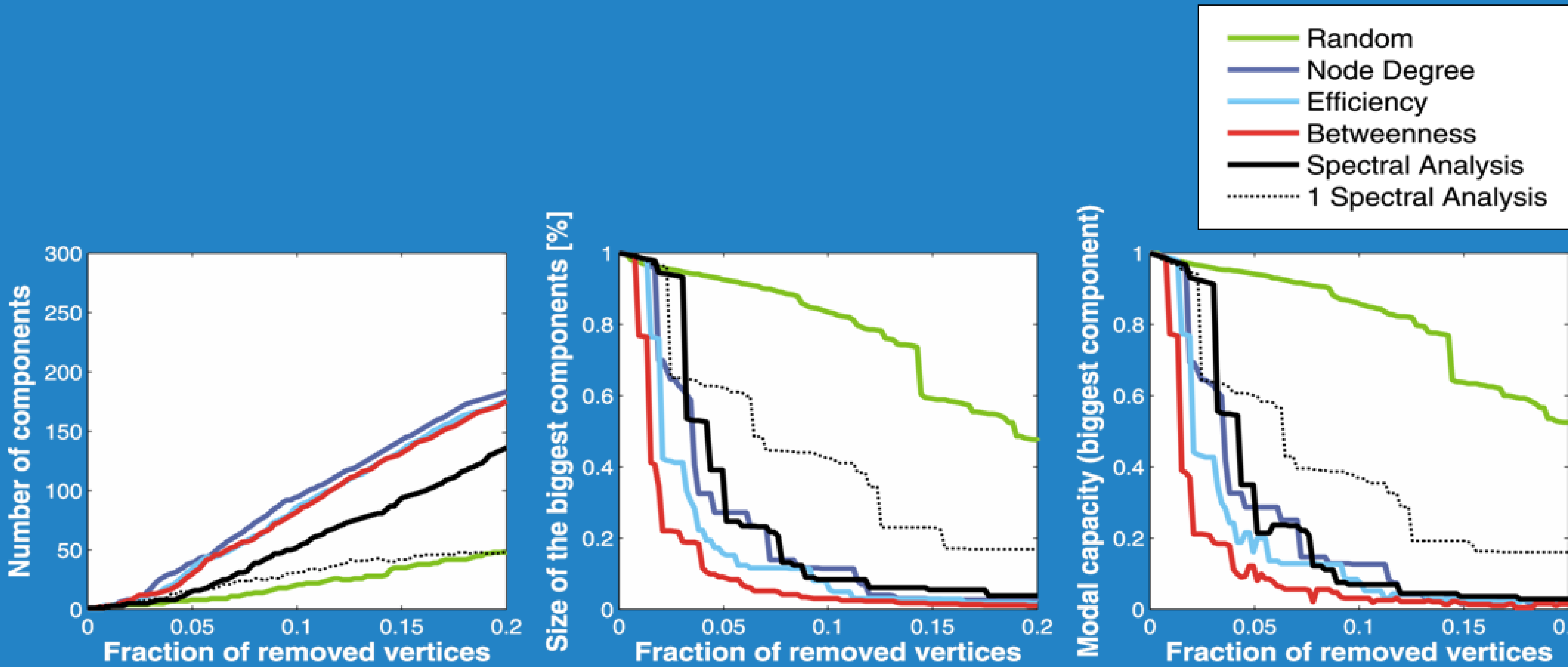


UCTE(UNION COORD TRANS ELECTRICiTY)

Algorithm of attacking the network

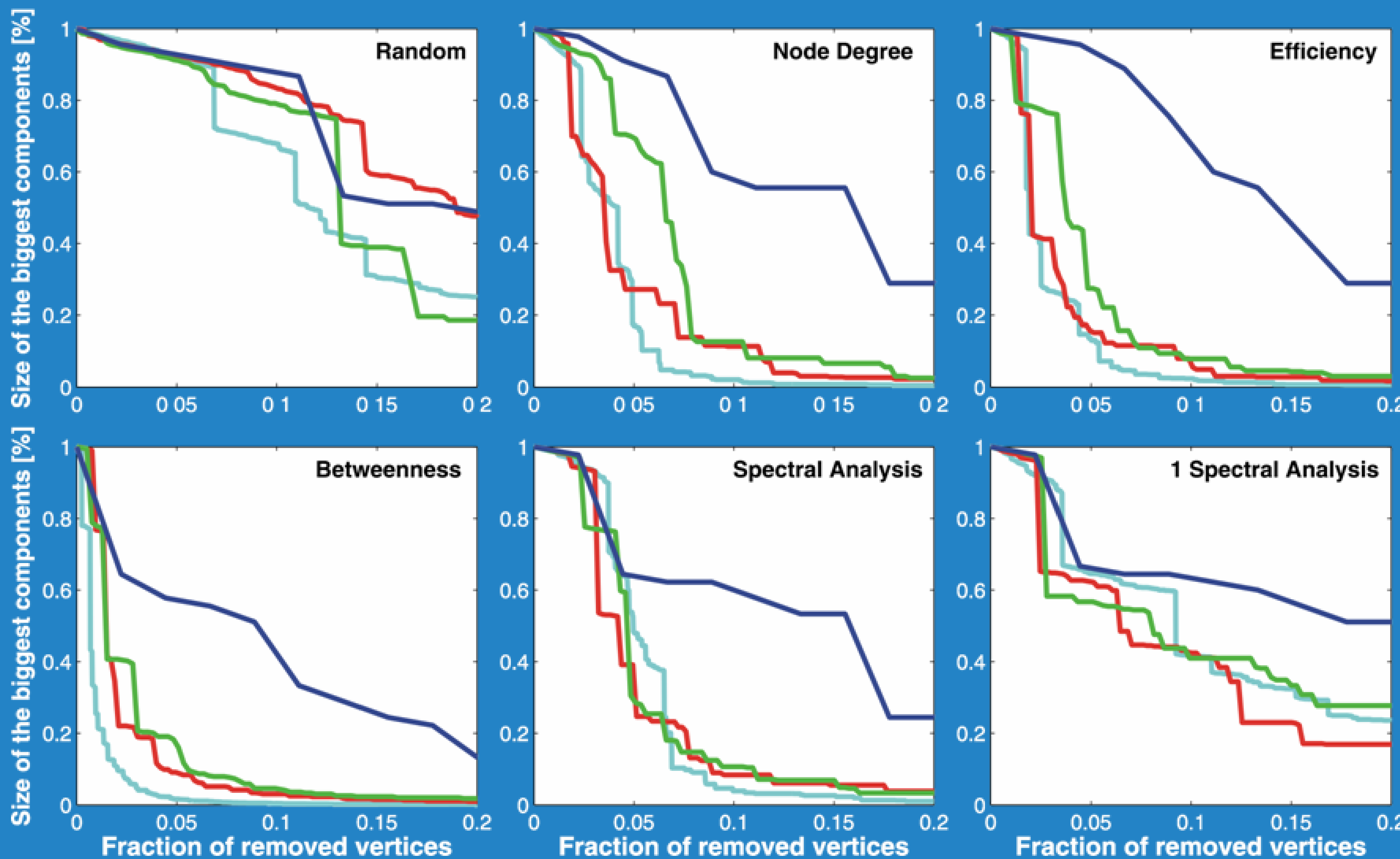


Measuring the consequences

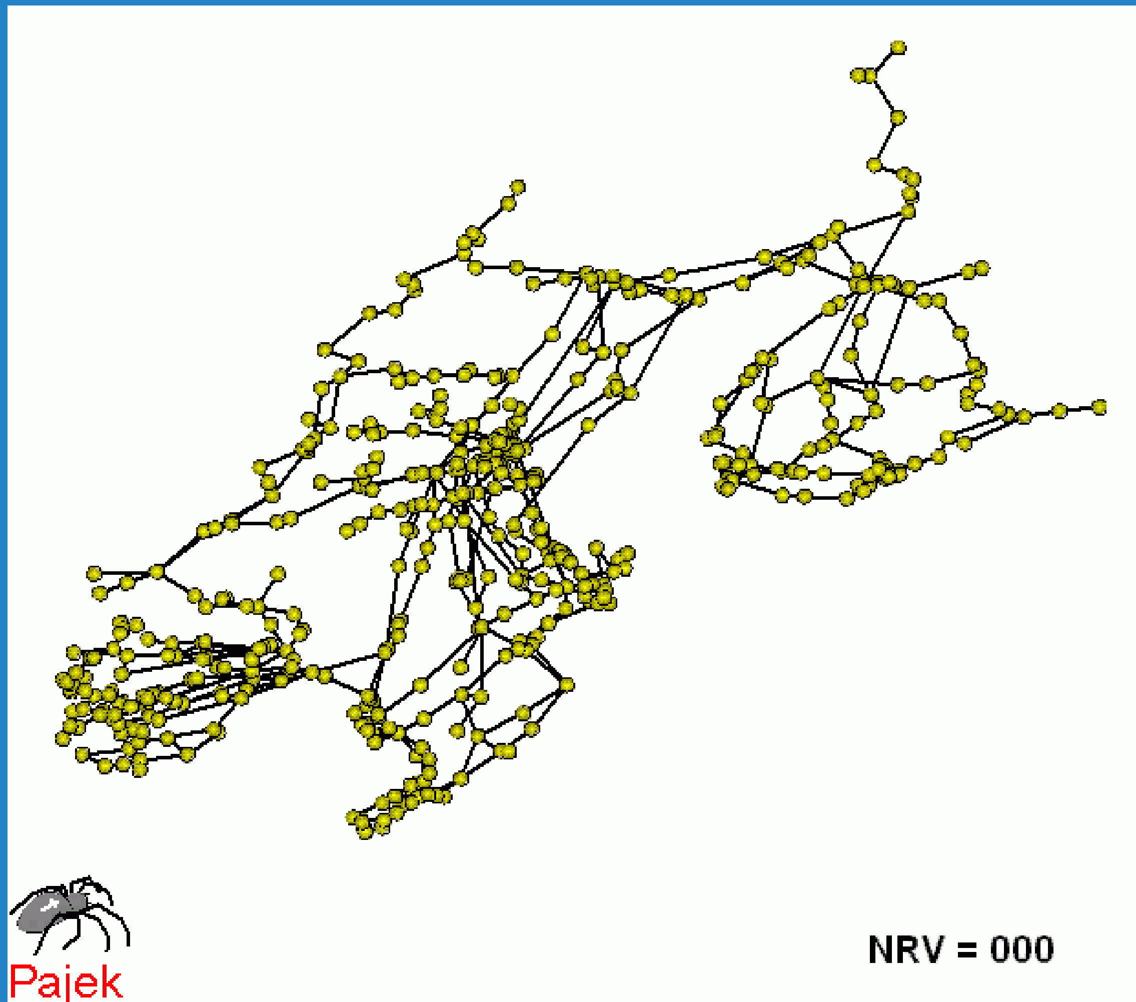


NORDEL

Measuring the consequences



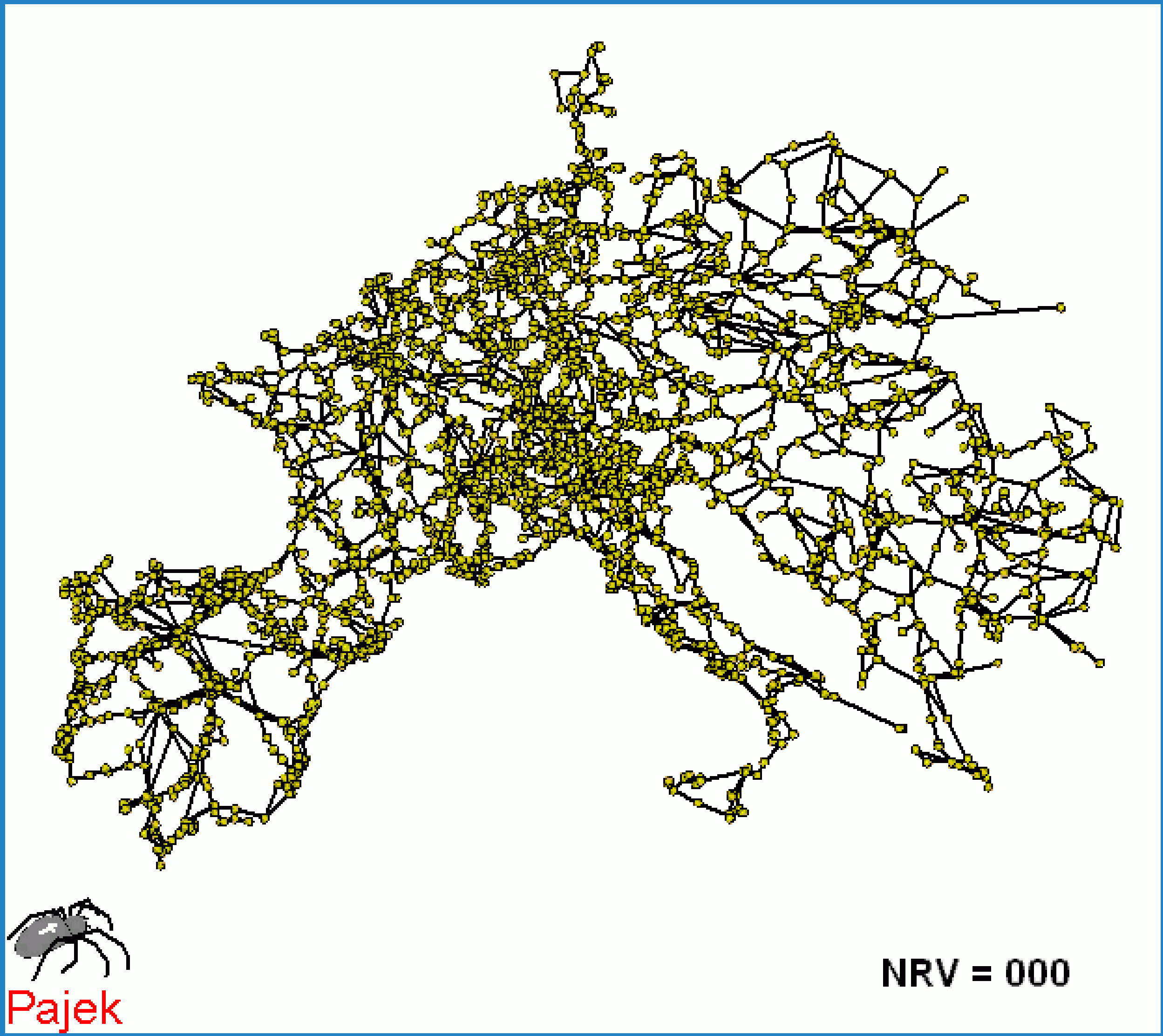
BREAK-UP OF THE NETWORK



Betweenness centrality NORDEL

NRV – number of removed vertices

BREAK-UP OF THE NETWORK



Betweenness centrality UCTE

NRV – number of removed vertices

FASTEST DECAY OF NETWORK

SIMULATION OF THE NETWORK ATTACK

- Removal of random nodes according to a ranking
- Decay is broadly comparable for the different types of status BC, Eff, SA
- Fastest first level fragmentation is by using the measure –

BETWEENNESS CENTRALITY

- other higher level fragmentation needs to be considered

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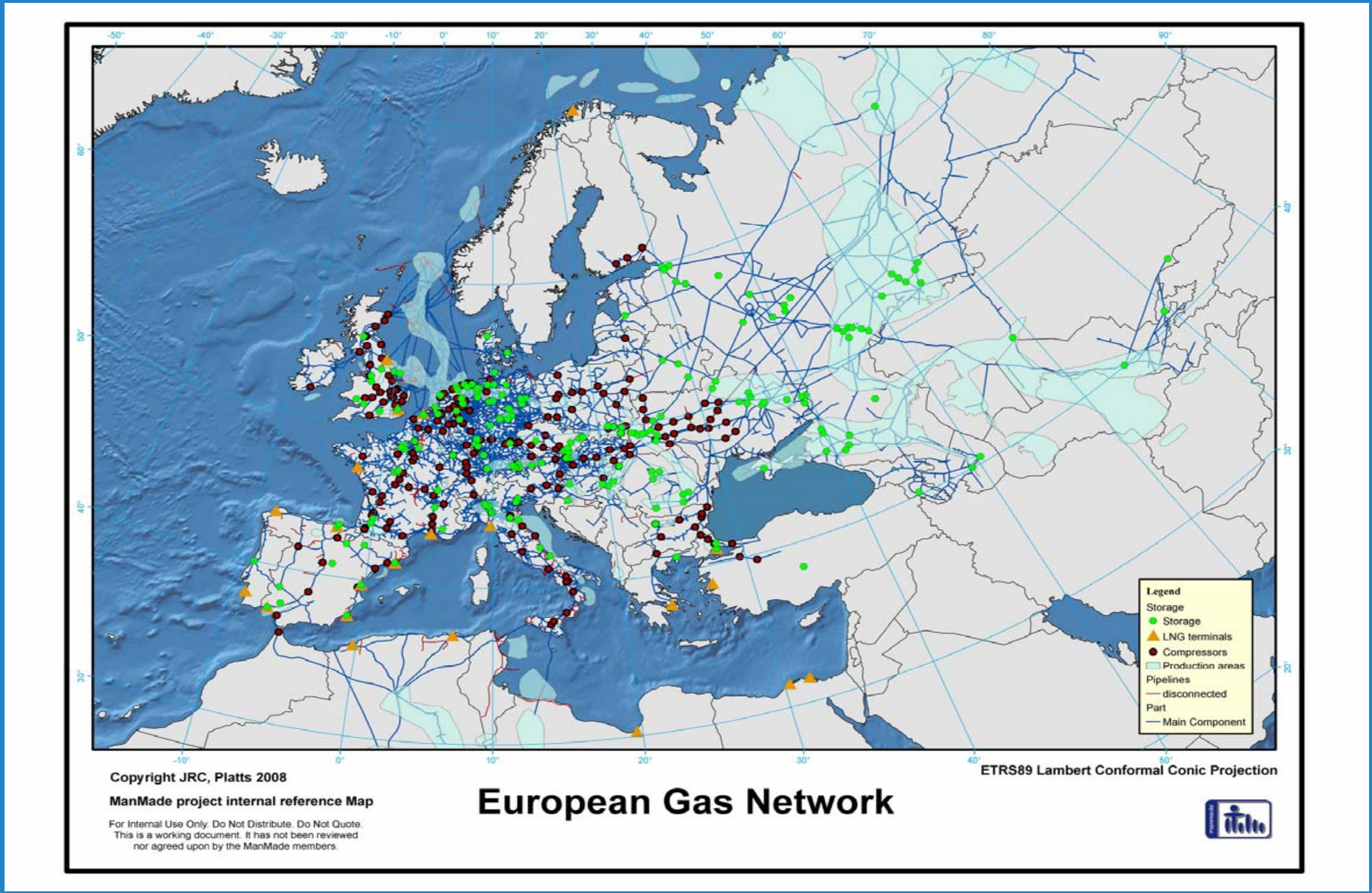
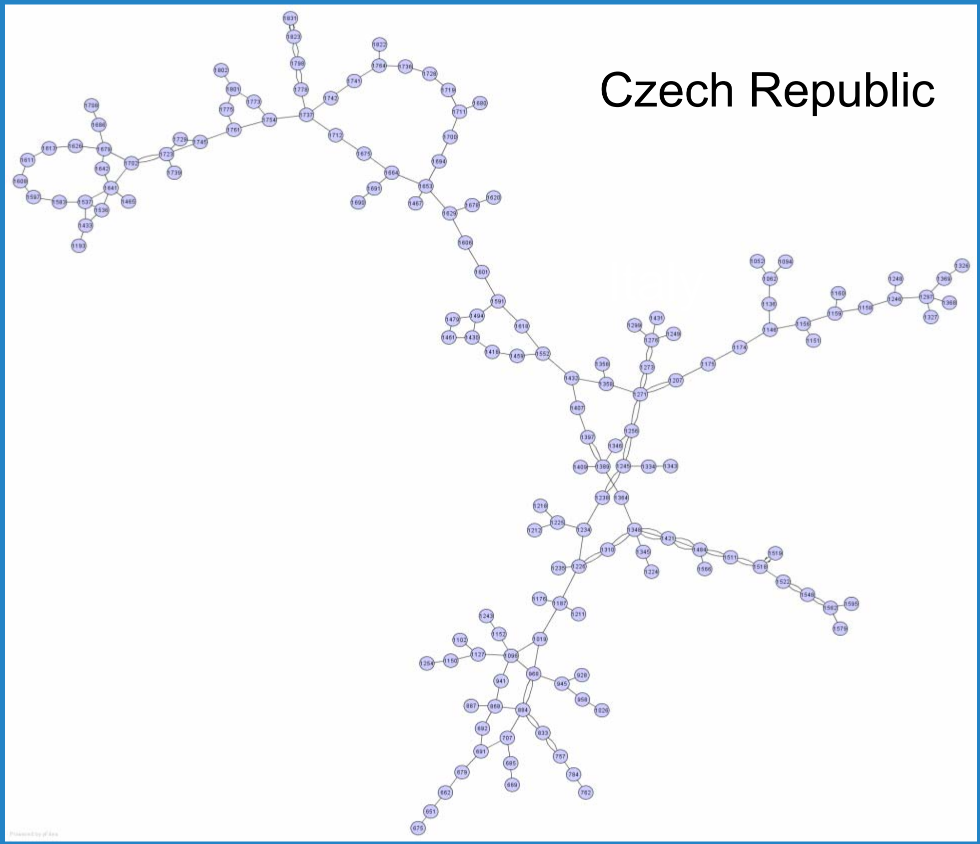
European Gas Network (JRC, Platts)



Gas Pipeline Network Layouts



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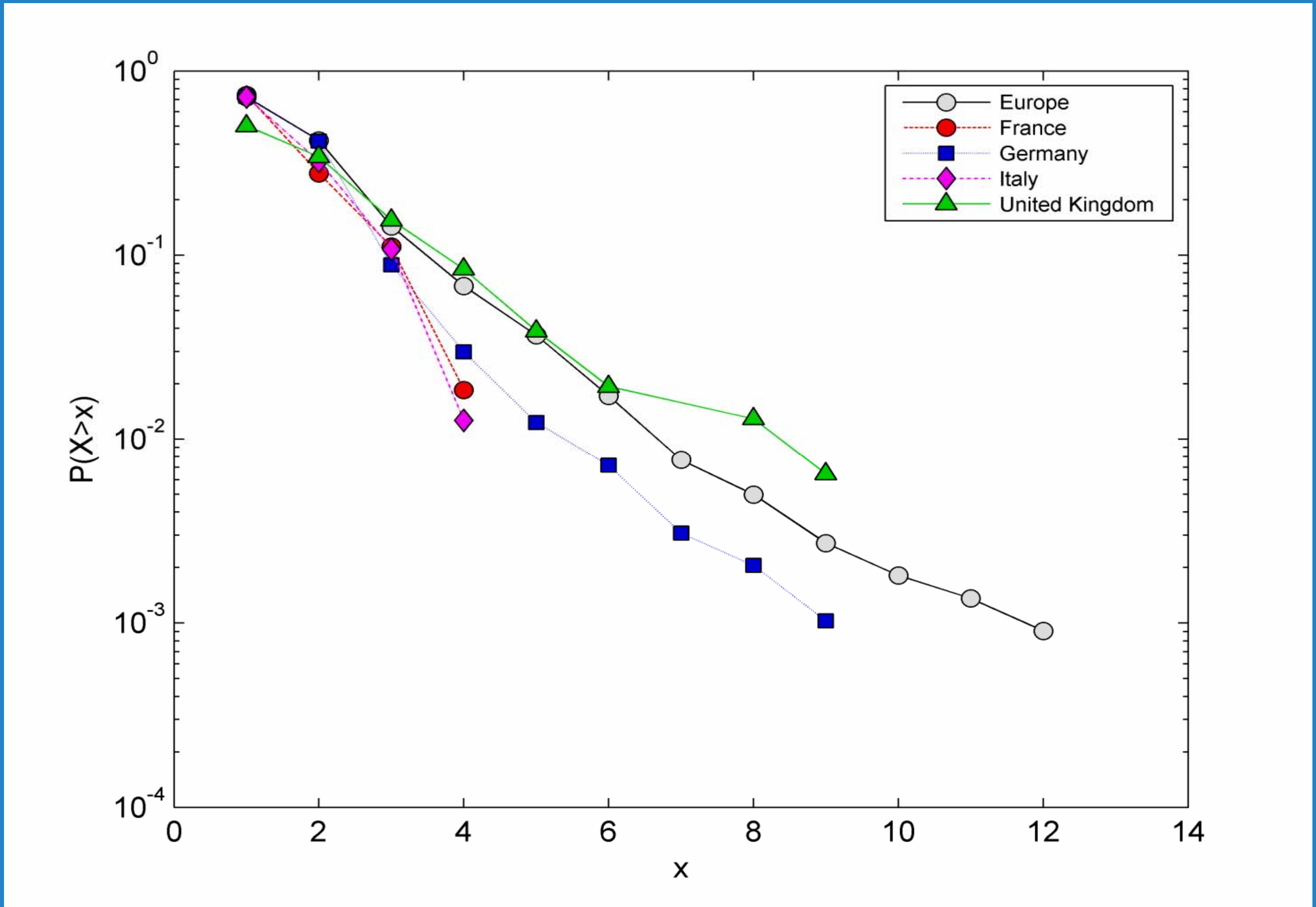
Gas Pipeline Network: Cumulative Distribution of Node Degree



x: node degree
P(X>x): probability of node degree in excess of x
Exponential decay

Network Motifs: Motivation

R Carvalho, QMUL

- Basic idea: to consider the recurring subgraphs of interactions from which networks are built.
- **Motif** (intuitive definition): Consider a "real world" network G.

  H is a subgraph of a given graph G iff H is a graph whose vertices and edges form subsets of the vertices and edges of G.
- A subgraph H of G is a ***motif*** if the number of appearances of H in the real network exceeds the average number of appearances of H in a random network ensemble.

- Claim: "real world" networks are organized in superfamilies according to their motifs.

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Network Motifs: Formal Definition

Milo et al. Superfamilies of Evolved and Designed Networks, *Science* **303**, 1538

- First, compute the abundance of each subgraph *i* relative to its appearance in random networks:

$$\Delta_i = \frac{N_{real} - \langle N_{rand} \rangle}{N_{real} + \langle N_{rand} \rangle + \varepsilon}$$

where a positive ε insures that $|\Delta|$ is not misleadingly large when the subgraph appears very few times in both the real and random networks;

- Normalizing gives *subgraph ratio profile* is defined as

$$SRP = \frac{\Delta_i}{\sum \left(\Delta_i^{\,2}\right)^{1/2}}$$

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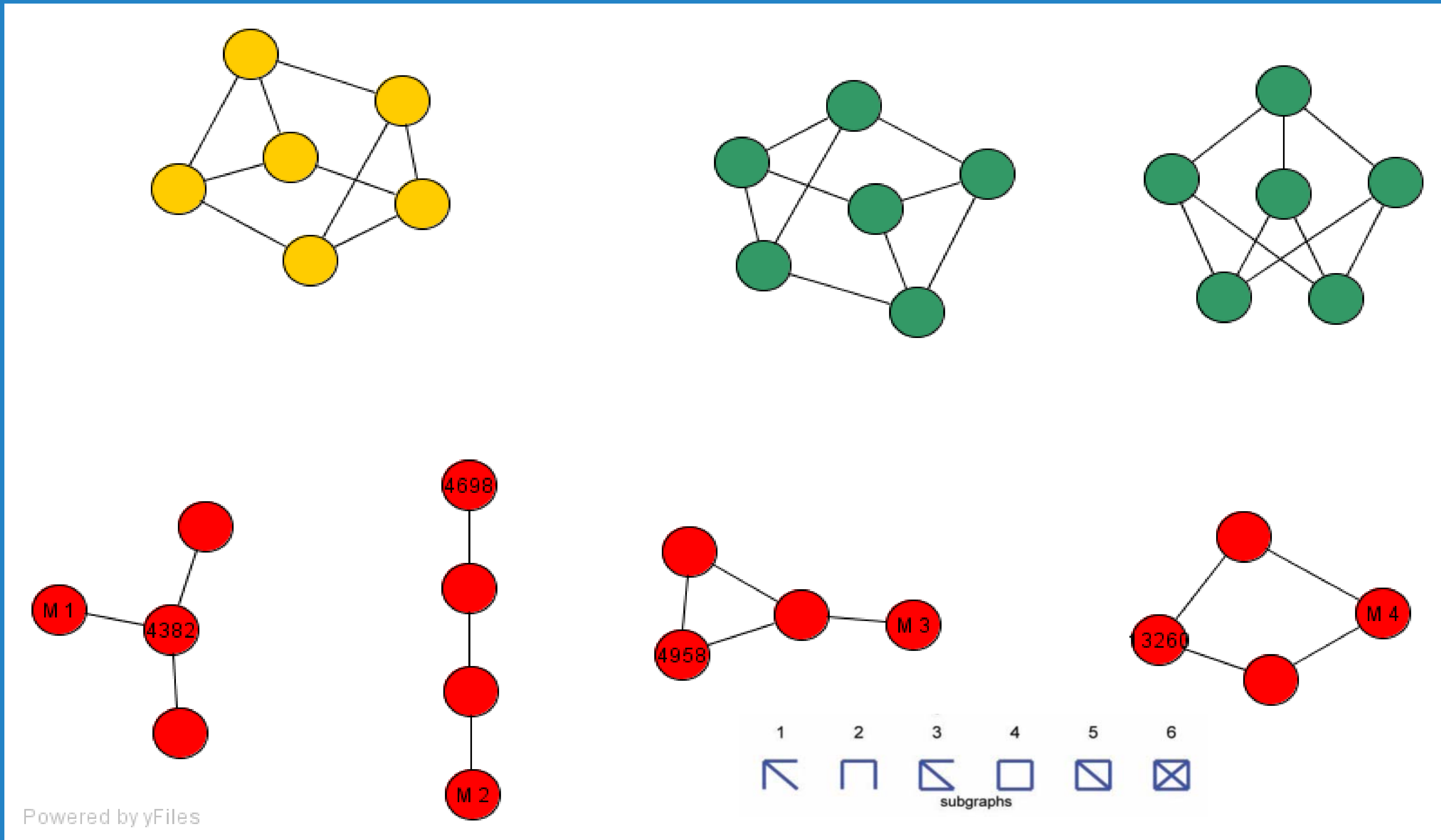
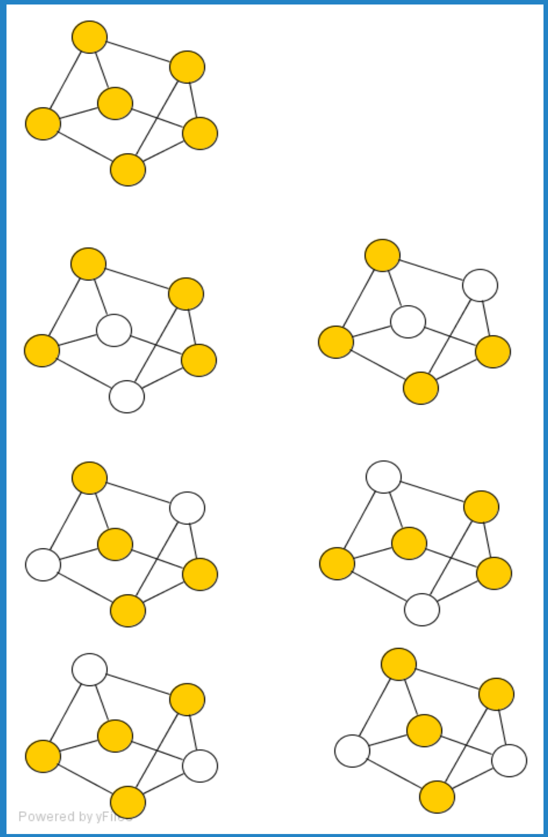
Network Motifs: Example

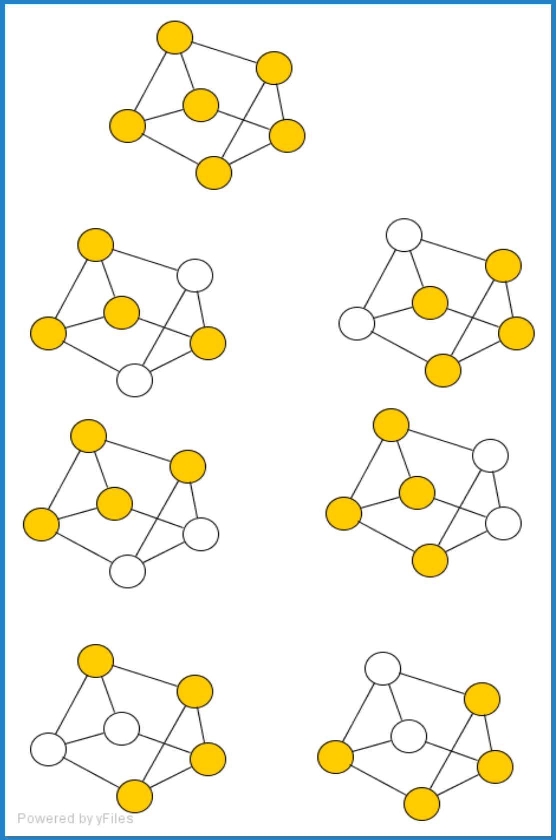


Network Motifs: Example

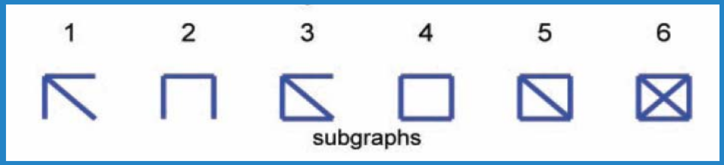
Presence of sub-graphs 2 and 3



Subgraph #2



Subgraph #3



Network Motifs: Superfamilies of Networks

Milo et al., Superfamilies of Evolved and Designed Networks, *Science* **204**, 1538

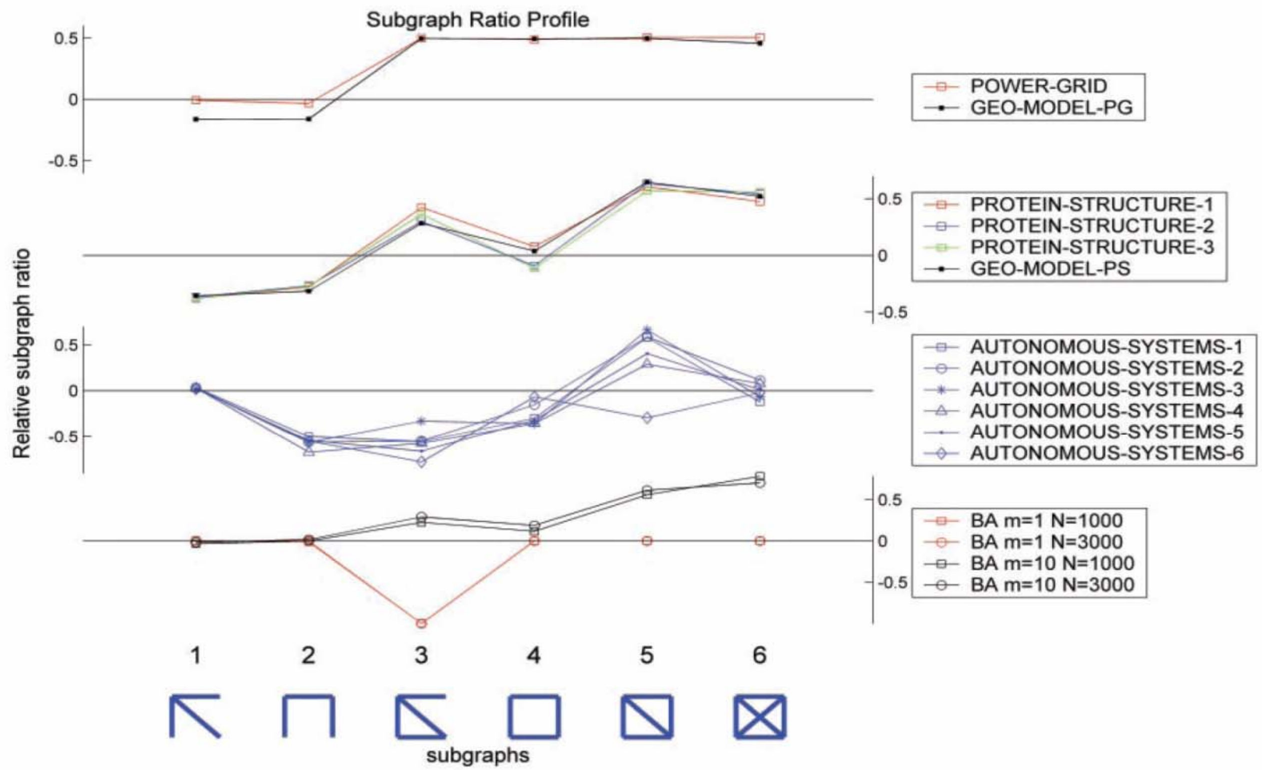
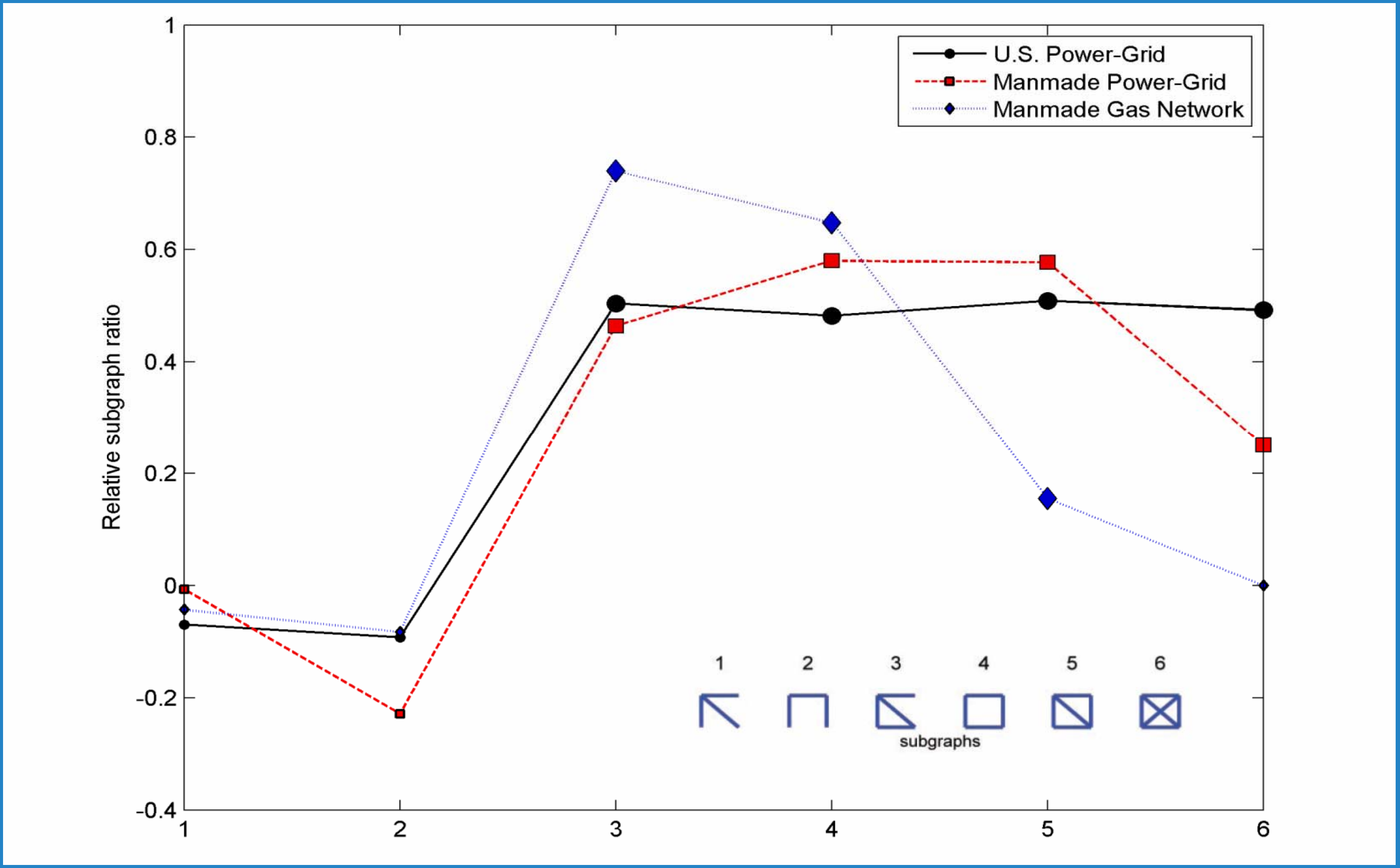


Fig. 3. The subgraph ratio profile (SRP) for various nondirected networks. The networks are as follows (*12*): (i) The electrical power grid of the western United States (*4*) (POWERGRID N = 4941, E = 6594) and a geometric model with similar clustering coefficient (GEO-MODEL-PG N = 5000, E = 7499). (ii) Networks of secondary-structure elements adjacency for several large proteins [structure based on the PDB database (www.rcsb.org/pdb/); the proteins (and their PDB ID) were 1A4J, an immunoglobulin (PROTEIN-STRUCTURE-1 N = 95, E = 213); 1EAW, a serine protease inhibitor (PROTEIN-STRUCTURE-2 N = 53, E = 123); and 1AOR, an oxidoreductase (PROTEIN-STRUCTURE-3 N = 99, E = 212)] and a geometric model with similar clustering coefficient (GEO-MODEL-PS N = 53, E = 136). (iii) The Internet at the autonomous system level (www.cosin.org) (AUTONOMOUS-SYSTEMS 1 to 6; N = 3015, 3522, 4517, 5357, 7956, 10515; E = 5156, 6324, 8376, 10328, 15943, 21455). (iv) Networks grown according to the preferential attachment BA model (*3*) with m = 1 or m = 10 edges per new node (BA m = 1, 10; N = 1000, 3000, 1000, 3000; E = 1000, 3000, 9901, 29901).

Motifs in manmade networks



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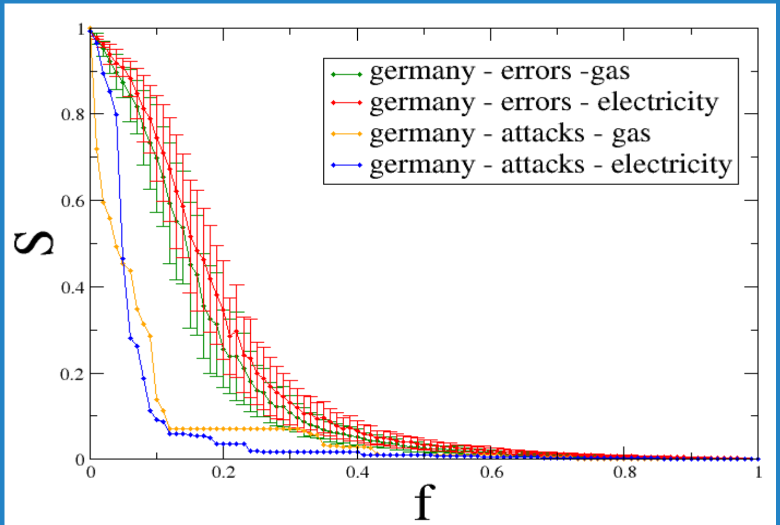
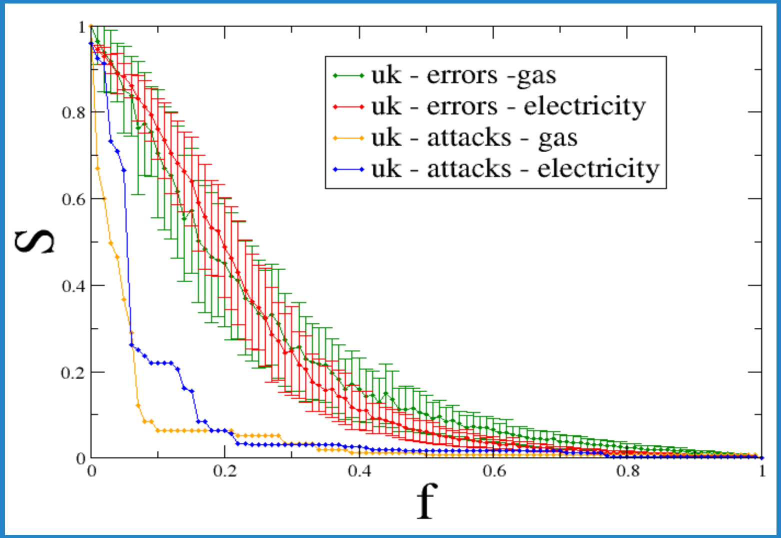
Network Tolerance Against Node Removals

We study the size of the largest connected component (S) as a function of the fraction of nodes removed (f) by:

- Errors: random node removal;
- Attacks: higher degree nodes are removed first;





Wind field construction

and maps of potential wind energy production over Europe

P. Kiss, I Janosi M. Szenes, Farkas with JRC(Ispra) support
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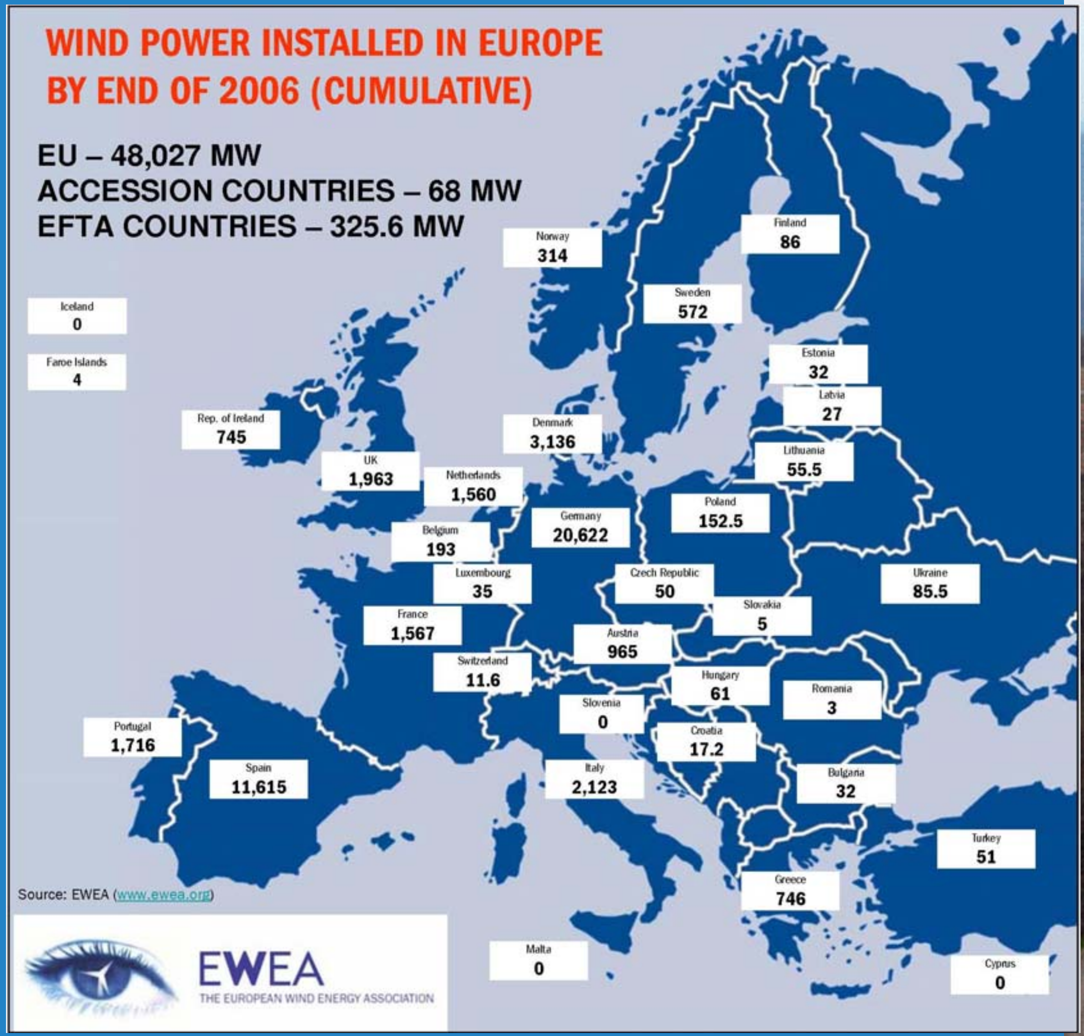
Data and methods

- Models for wind speed histograms
- Wind profiles – height dependence of wind speed
- Wind power estimations
- Wind power networks

Wind field construction

and maps of potential wind energy production over Europe



wind energy

as a %

of total energy

in EU

7-8%

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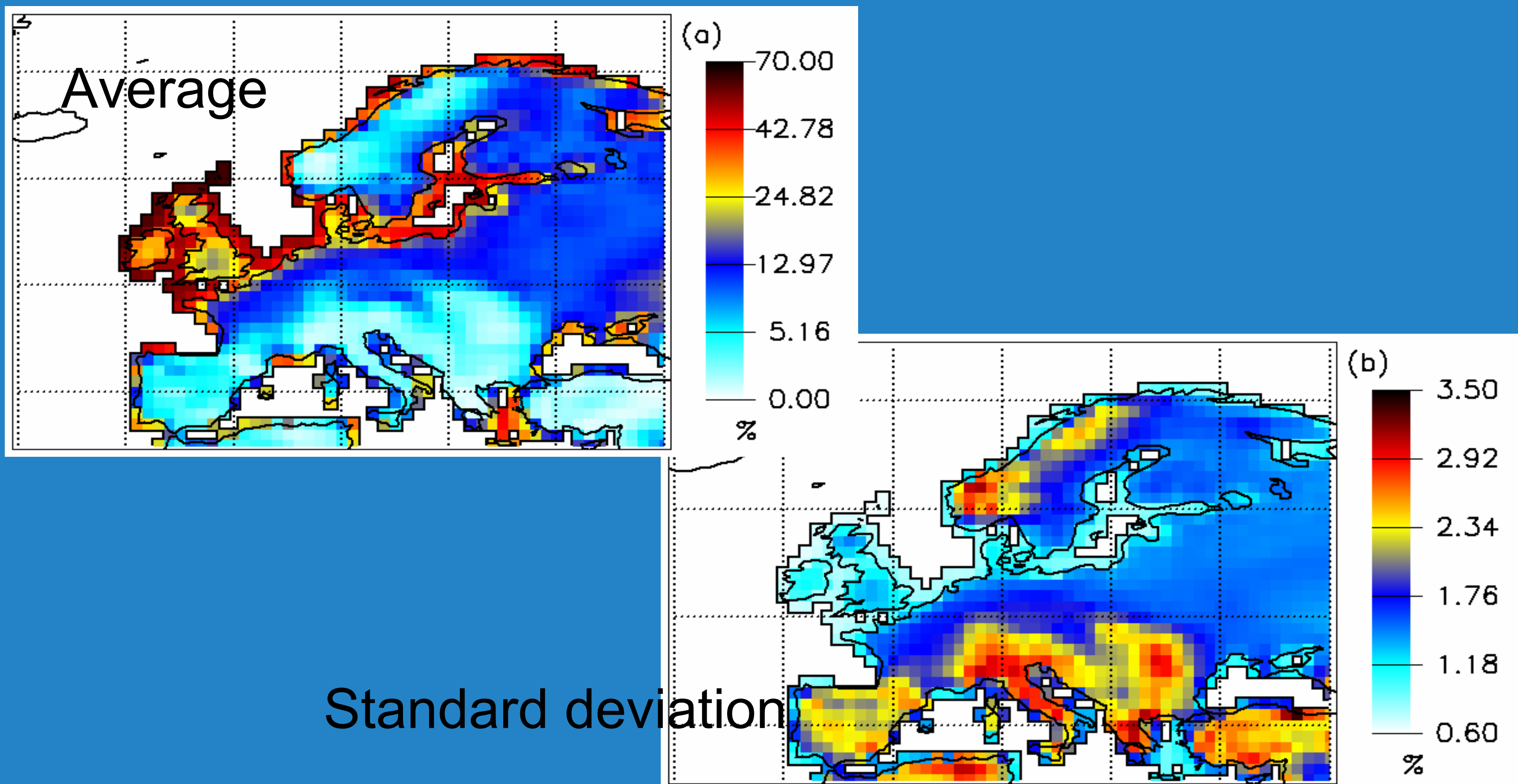
Wind power networks

Average and standard deviation of wind power



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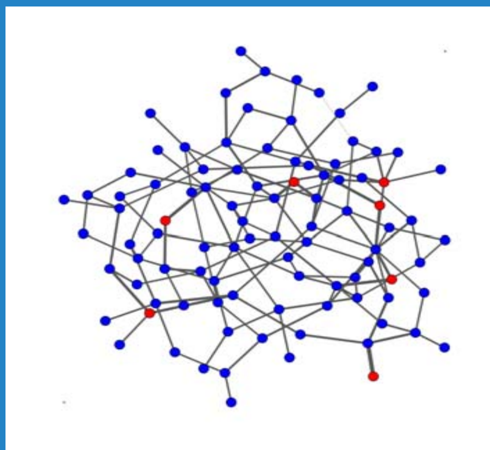
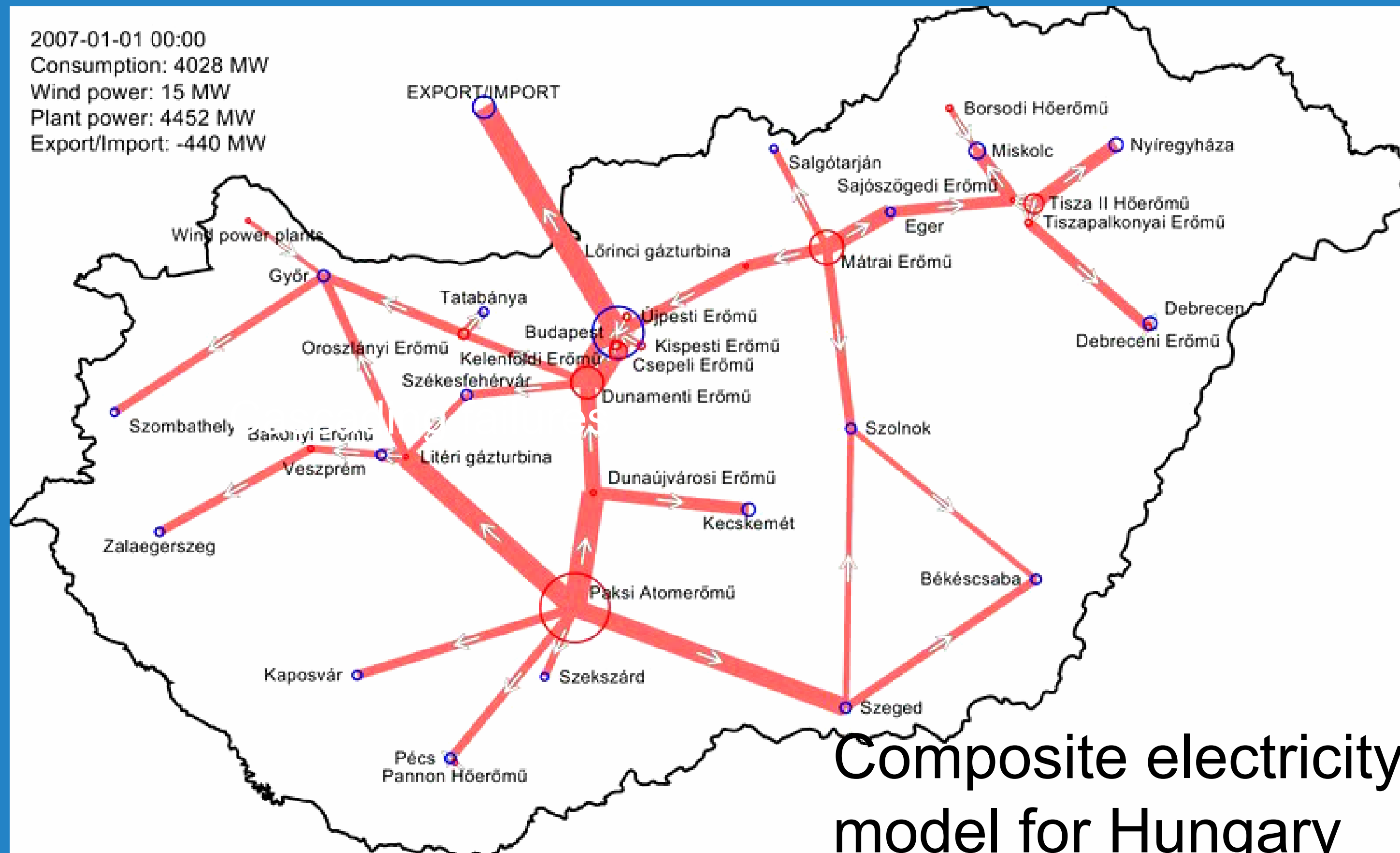
Error tolerance of complex networks





Cascading breakdown
flow of a physical quantity:
maximal load is limited by the capacity of the edge
edge removal leads to redistribution of the initial loads -overloading

Composite electricity model for Hungary including wind energy

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Error tolerance of complex networks

Dynamic network capacity modelling

- extend the model to the whole European power grid network using available European grid topology and REWIRING on the basis of known offshore windspeed data
  - emergence of a new network topology incorporating windpower
- identifying the most vulnerable parts of the network –
- suggesting new edges (transmission lines) which make the network more tolerant

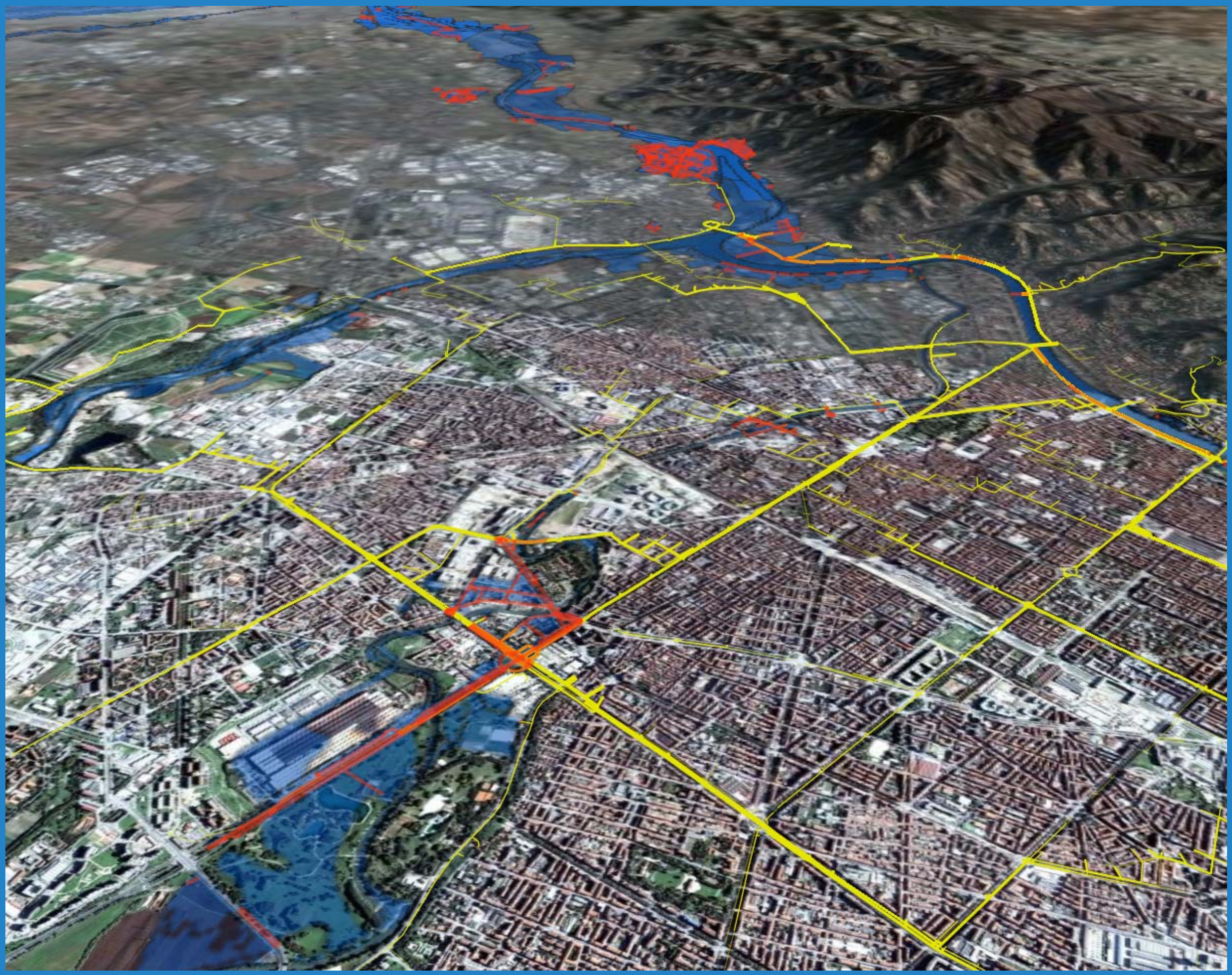
Urban Networks

- Milan
- Turin
- London (in progress)

Data Sources:

- TeleAtlas
- UK DfT
  Department for Transports
- Civil protection surveys

Identification of vulnerable transport infrastructures



Topological measures of damaged and undamaged network.

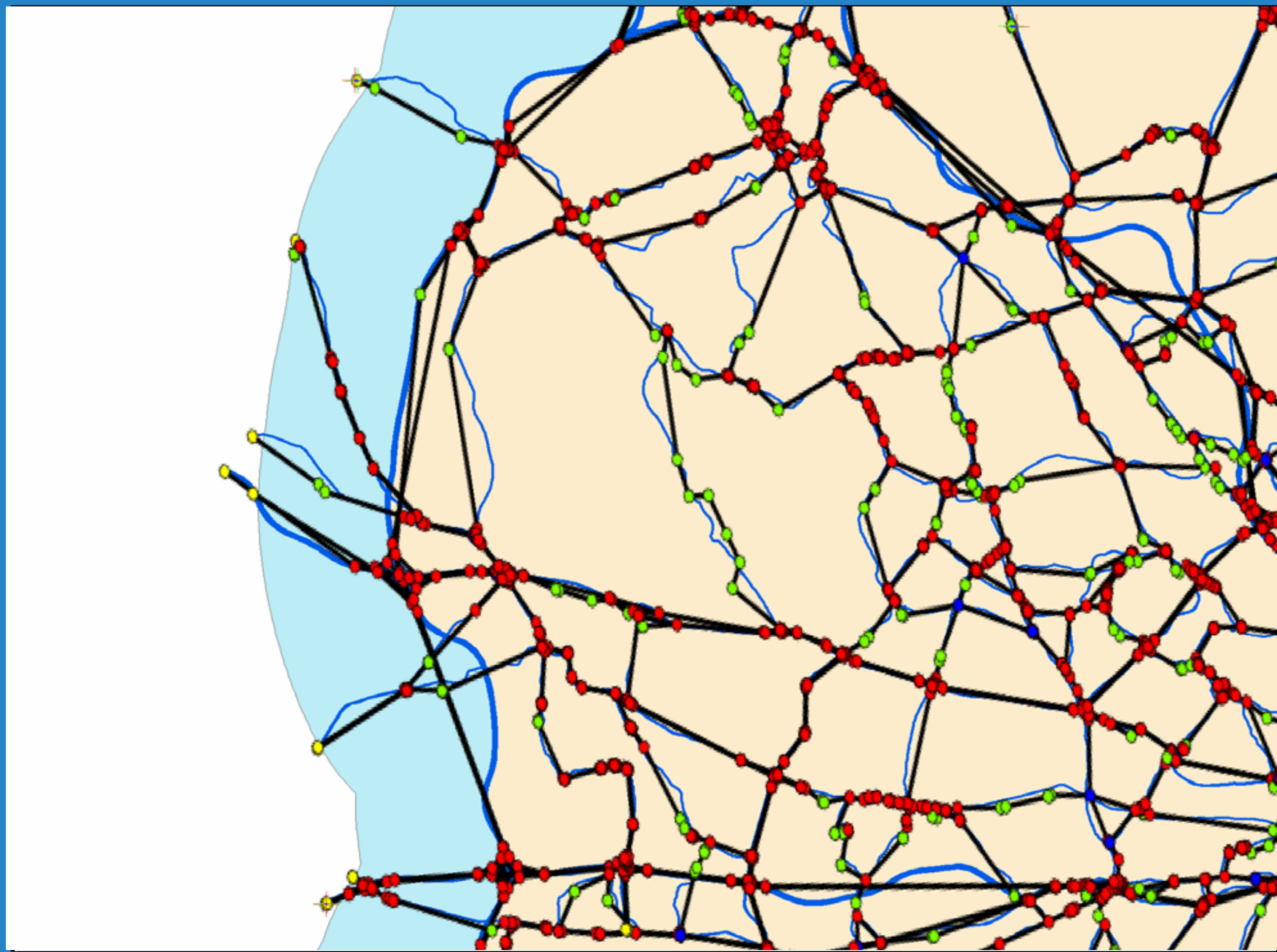
Structural vulnerability, key transport nodes, planning and **protection.**

Centrality
is a topological measure of connectivity rank.

Flooded area
(Turin - October 2000).

Intersection of flood and high centrality (orange).

Urban Traffic

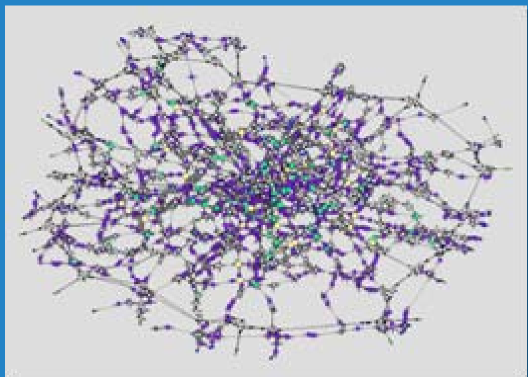


AADTF – annual average daily flow

Network simplification

Connectivity analysis

Future datasets

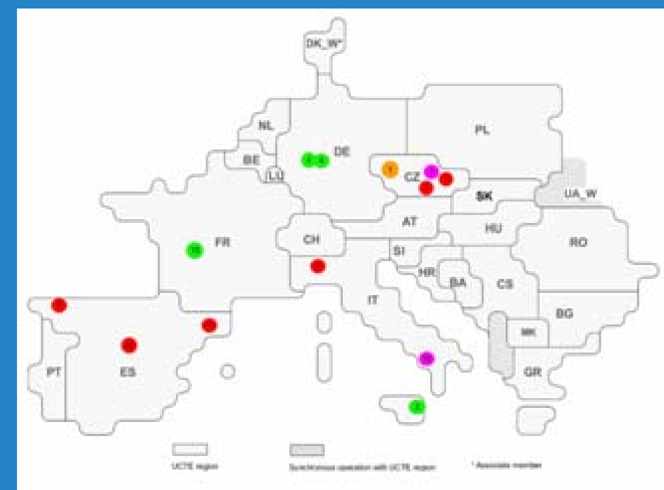


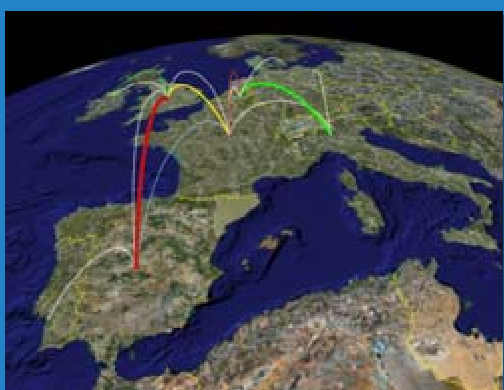
Urban street network of London and traffic counts

2232 counters, counts from 1999 to 2006, 8566 nodes, 15573 arcs

Electricity Network disruptions

overlay on ranking networks for correlation





Commodity Flows

27 countries, 6 Major groups,
225 products, years 2005-2007

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Dynamic of London Traffic (© Manmade)