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T. von Hippel, C. Simpson, N. Manset

On the Accretion of Distant Planets

William R. Ward

Department of Space Studies, Southwest Research Institute, 1050 Walnut Street, Boulder, CO 80302, USA

Craig B. Agnor

Department of Space Studies, Southwest Research Institute, and Department of Physics, University of Colorado, Boulder, CO 80302, USA

Hidekazu Tanaka

Department of Space Studies, Southwest Research Institute and Department of Earth and Planetary Sciences, Tokyo Institute of Technology, Tokyo 152-8551, Japan

Abstract. We consider the accretion time scale of planetary objects at large stellar distances such as Neptune. The shallow stellar potential well inhibits the formation of large, distant companions in three important ways: (1) orbital periods are longer, and so more remote regions are dynamically younger; (2) the volume occupied by accreting material is larger, contributing to low collision frequencies; and (3) the low gravitational binding energy to the star makes it easier for a planetary embryo to scatter planetesimals out of the system. We suggest that resonant damping of embryo dispersion velocities by a dissipating remnant of their precursor gas disk may mitigate this situation and make the existence of large, outer planets like Neptune easier to explain. Disk torques dynamically cool a system of embryos at the expense of more remote regions of the gas disk by driving acoustic waves at coorbiting Lindblad resonance sites. This lowers embryo eccentricities and inclinations; the smaller dispersion velocities increase the embryos' collision cross sections, which in turn shortens accretion times and results in less material ejected from the system. If the gas surface density has decreased to the point in which it is comparable to that of the condensed solids, the embryo's equilibrium eccentricities become comparable to the normalized scale height of the gas disk. In such a case, the disk torques causing orbit migration are significantly weakened and the protoplanet could be relatively stable against type I decay.

1. Introduction

A long standing puzzle regarding the formation of the Solar System concerns the accretion of the outermost planets, Uranus and Neptune. Despite successes in modeling the formation of the terrestrial planets, these same concepts seem to fall short for the most remote planets of our Solar System. For one thing, because orbital periods P in a Keplerian disk are longer at large heliocentric distances ($P \propto a^{3/2}$, where a is the semi-major axis) such regions are dynamically younger. Secondly, the volume of space occupied by planetesimals is so large that collision frequencies are very low—leading to accretion time scales that seem to be unacceptably long, i.e., greater than the age of the Solar System (Lissauer et al. 1995; Levison & Stewart 2001). A third issue relates to the low stellar binding energy of material in distant parts of the disk. This results in large orbital eccentricities $e \sim v/a\Omega$ (where Ω is the mean motion) for a given planetesimal dispersion velocity, v. If a planetary embryo of mass M, radius R, and body density ρ, becomes large enough that its escape velocity, $v_{\rm esc} = \sqrt{2GM/R} = R\sqrt{8\pi G\rho/3}$, exceeds the escape velocity from the star, $\sqrt{2GM_\star/r} = \sqrt{2}r\Omega$, it becomes a very efficient scatterer and can eject much of the material out of the system before it can be accreted. This occurs for embryo masses greater than

$$M/M_\star = \sqrt{3M_\star/4\pi\rho r^3} = 4\times10^{-4}\rho_{\rm cgs}^{-1/2}(r/{\rm AU})^{-3/2}\sqrt{M_\star/M_\odot}, \qquad (1)$$

which is only a fraction of an Earth mass in the outer Solar System beyond the orbital distance of Uranus, $r \geq 20$ AU. The masses of Uranus and Neptune exceed this threshold by a significant margin. Indeed, their scattering properties are implicated in the formation of the Oort cloud and the scattered disk component of the Kuiper belt (Duncan & Levison 1997; see also Malhotra, Duncan, & Levison 2000, and references therein).

Here we explore the effect of a small remnant of the gas disk on the accretion process. The H/He content of Neptune is only a few percent that the gas giants. If this is indicative of a reduced, but non-zero gas density in that region during the final assembly of this planet, its presence can have a surprisingly important influence on the dynamics of forming embryos. Resonant interaction of the embryos with the tenuous gas disk will damp embryo dispersion velocities to some fraction of the gas sound speed, largely independent of their mass (Ward 1993). This allows for a significant enhancement in the collision cross section (by a factor $F_{\rm g} \sim 1 + (v_{\rm esc}/v)^2 \sim$ few $\times 10^2$), and shortens the accretion time scale accordingly. We argue here that this behavior, coupled with fresh insight into the phenomenon of protoplanet migration at high eccentricities (Papaloizou & Larwood 2000) provides a new avenue for accretion of the outermost planets.

2. Standard Accretion Theory

The characteristic time scale of the growth of a planetary embryo from a disk of material of surface density $\sigma_{\rm d}$ about a solar mass star can be written

$$\tau_{\rm acc} \sim \frac{\rho R}{\sigma_{\rm d}\Omega}\left(\frac{c_{\rm acc}^{-1}}{F_{\rm g}}\right) = 3\times10^{10}\left(\frac{c_{\rm acc}^{-1}}{F_{\rm g}}\right)\left(\frac{M}{M_\oplus}\right)^{1/3}\left(\frac{r}{30{\rm AU}}\right)^{3/2}\left(\frac{\rho^{2/3}}{\sigma_{\rm d}}\right)_{\rm cgs} {\rm yr}, \quad (2)$$

where $M_\oplus = 6\times10^{27}$ g is an Earth mass and $c_{\rm acc}$ is a constant of order unity that depends in part on the I/e ratio of the planetesimals' orbital inclinations, I, and eccentricities, e, (e.g., Lissauer & Stewart 1993). If we set $M \sim 15M_\oplus$, $\rho \sim 1.64$ g cm^{-3} (the density of Neptune), $c_{\rm acc} \sim 1$, $r = 30$ AU, then Equation 2 reads, $\tau_{\rm acc} \approx 10^{11}\sigma_{\rm d,cgs}^{-1}F_{\rm g}^{-1}$ yr. The surface density is of order $\sigma_{\rm d} \sim M_{\rm d}/\pi r^2 \sim 10^{-2}(M_{\rm d}/M_{\oplus})(r/30{\rm AU})^{-2}{\rm g\ cm}^{-2}$; note to account for at least the ~ 17 Earth masses contained in Neptune, $\sigma_{\rm d} \gtrsim 0.2$ g cm^{-2}. The value of the enhancement factor $F_{\rm g}$ describes the increase in the cross section due to gravitational focusing and can affect the time scale estimate considerably. If the dispersion velocity v is due to mutual scattering among field particles of radius R', it will acquire a value on the order of their escape velocities, $v'_{\rm esc}$ (e.g., Safronov 1969) and $F_{\rm g} \approx (R/R')^2$. This results in $\tau_{\rm acc} \propto 1/R$ and the well-known accretion runaway behavior. Depending on the size differential, $F_{\rm g}$ could become quite large, although there are limits (e.g., Greenzweig & Lissauer 1990; Ida & Makino 1993).

A principle limiting factor is the influence of the target itself on the field particles' dispersion velocity (e.g., Lissauer 1987). The stirring action of the embryo generates a minimum dispersion velocity of order $v \sim c_{\rm H}R_{\rm H}\Omega$, where $R_{\rm H} \equiv r(M/3M_\star)^{1/3}$ is the embryo's Hill radius (e.g., Ida & Makino, 1993). This sets the maximum value for $F_{\rm g}$ at $F_{\rm max} \sim 6c_{\rm H}^{-2}(4\pi\rho r^3/3M_\star)^{1/3} \approx \hat{F} \times 10^3(r/30\ {\rm AU})$, which is independent of embryo size. Numerical experiments typically yield scaled velocities in the range $c_{\rm H} \approx 4$–6 (e.g., Tanaka & Ida 1997), so that the value of $\hat{F} \equiv 28/c_{\rm H}^2$ is likely of order unity. The target-to-field-particle size ratio at which this limit is reached is $R/R' \sim \sqrt{F_{\rm max}}$. Past that point, growth times are again proportional to R; this is the so-called oligarchic growth. Nevertheless, the rate remains fast enough to form an $M \sim 15$ Earth mass object in $\sim 10^8$ yr if it could continue unabated (e.g., Lissauer et al. 1995, see Figure 1).

However, unabated oligarchic growth is an important caveat because if $\tau_{\rm acc} \propto R$, there may well be time for other embryos to initiate runaway growth and close the size ratio. As competing embryos develop, they may exhaust the supply of field particles prematurely. Their mutual perturbations then begin to raise the general dispersion velocity, and the enhancement factor diminishes (e.g., Iwasaki et al. 2001). An embryo population undergoes gravitational relaxation at a rate of order (e.g., Stewart & Wetherill 1988)

$$\tau_{\rm scatt} \sim \frac{c_{\rm scatt}^{-1}}{\Omega}\left(\frac{M_\star}{M}\right)\left(\frac{M_\star}{\pi\sigma_{\rm d}r^2}\right)\left(\frac{v}{r\,\Omega}\right)^4, \tag{3}$$

where $c_{\rm scatt} \approx 0.4\ln[(M_\star/M)(v/r\Omega)^3]$ is a constant of order a few, and collisional damping at a rate of order Equation 2. This implies that a population in collisional equilibrium tends to relax to a dispersion velocity comparable to the individual escape velocities of the embryos (e.g., Safronov 1969). Again, this results in: (1) an enhancement factor of order unity; (2) frequent scattering out of the system (if $v_{\rm esc} > r\ \Omega$); and (3) a prohibitively long accretion time for the outermost planets (shown by the ordered growth curve in Figure 1).

3. Disk Damping

If the embryos are embedded in a gas disk of surface density, $\sigma_{\rm d}$, and a gas sound speed, c, another damping mechanism operates involving wave interactions at

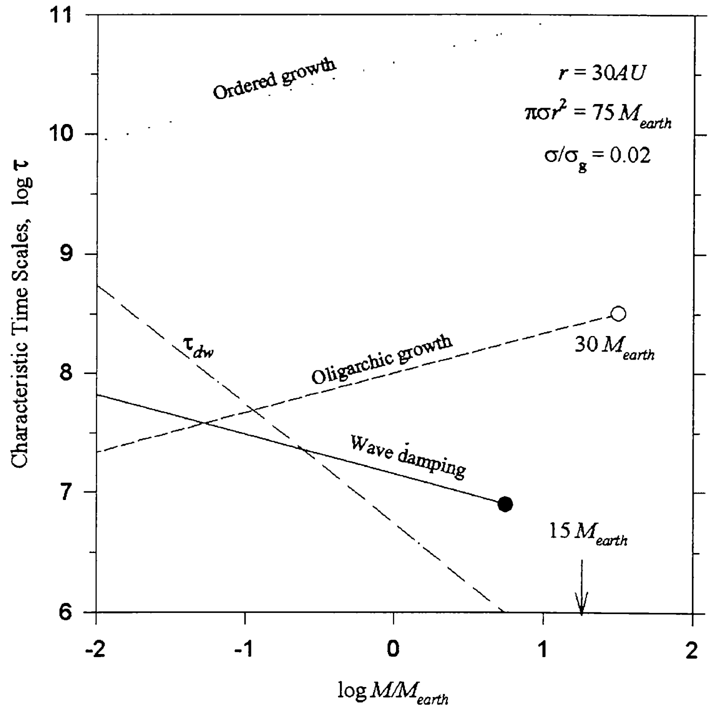

Figure 1. Comparison of characteristic growth times $\tau \equiv M/\dot{M}$ for three styles of accretion: ordered, oligarchic, and wave assisted growth. Also shown is the characteristic orbital decay time $\tau_{\rm dw} \equiv r/\dot{r}$ due to disk torques. Solid to gas ratio is taken to be $\sigma_{\rm d}/\sigma_{\rm g} = 0.02$.

"coorbiting" Lindblad resonances (Ward 1993, 1988a, 1988b; Artymowicz 1993). The characteristic time scale for this process is

$$\tau_{\rm dw}^{e} \sim \frac{c_{\rm e}^{-1}}{\Omega}\left(\frac{M_\star}{M}\right)\left(\frac{M_\star}{\pi\sigma_{\rm g}r^2}\right)\left(\frac{c}{r\Omega}\right)^4, \tag{4}$$

where $c_{\rm e}$ is a constant of order unity. Equating this to the scattering time scale [Equation 3] provides a new estimate for the dispersion velocity, viz.,

$$v \approx c_{\rm dw}(\sigma_{\rm d}/\sigma_{\rm g})^{1/4}c, \tag{5}$$

implying the embryos attain a mildly subsonic velocity in the gas independent of their size (Ward 1988b, 1993); a similar result has also been recently obtained both analytically and numerically by Papaloizou & Larwood (2000). In Equation 5, $c_{\rm dw} \equiv (c_{\rm scat}/c_{\rm e})^{1/4}$ is also of order unity and relatively insensitive to uncertainties in the other proportionality constants by virtue of its weak power dependence; henceforth we set $c_{\rm dw} = 1$. For example, in a solar composition gas, the abundance of solid material is only a small fraction of the gas, i.e., $\sigma_{\rm d}/\sigma_{\rm g} \approx 0.02$, so that $v \sim 0.4c$. Since the gas disk is thin, with a scale height $h \sim c/\Omega \ll r$, we have $v \lesssim c \ll r\Omega \ll v_{\rm esc}$, the last inequality being true for embryos larger than Equation 1. The damping furnished by the gas disk prevents the enhancement factor from decreasing to order unity if and when the oligarchic phase of accretion breaks down. The corresponding enhancement factor using Equation 5 reads

$$F_{\rm g} \sim 1 + (v_{\rm esc}/c)^2 \sqrt{\sigma_{\rm g}/\sigma_{\rm d}}. \tag{6}$$

Assuming $v_{\rm esc} \gg c(\sigma_{\rm d}/\sigma_{\rm g})^{1/4}$, the characteristic accretion time scale becomes

$$\tau_{\rm acc} \sim \frac{3c_{\rm acc}^{-1}}{8\Omega}\left(\frac{r}{R}\right)\left(\frac{M_\star}{\pi\sigma_{\rm d}r^2}\right)\left(\frac{c}{r\Omega}\right)^2 \sqrt{\sigma_{\rm d}/\sigma_{\rm g}}. \tag{7}$$

We obtain $\tau_{\rm acc} \approx 1\times 10^{10} c_{\rm acc}^{-1}\sigma_{\rm d,cgs}^{-1}(M_\oplus/M)^{1/3}(c/r\Omega)^2\sqrt{\sigma_{\rm d}/\sigma_{\rm g}}$ yr for the values employed here, which is shown in Figure 1 for the solar composition gas/solid ratio. Consequently, a 15 $M_\oplus$ object in a gas disk with a normalized scale height of $c/r\Omega \sim O(10^{-1})$ has a characteristic accretion time scale of order $\tau \approx 4\times 10^7 \sigma_{\rm d,cgs}^{-1}\sqrt{\sigma_{\rm d}/\sigma_{\rm g}} \approx 6\times 10^6$ yr. Note however, that $\tau \propto 1/R$ so that more of the time is expended at the smaller sizes. This enhanced accretion rate due to dispersion velocity damping by a gas disk was first described by Ward (1993), but two provisos must be reiterated: (1) the system should not be over-damped to the degree that individual embryos become isolated on non-crossing orbits[1]; and (2) the system must survive against orbital decay due to disk torques over the accretion time scale.

The first criterion can be satisfied by requiring $v/\Omega \gtrsim M/2\pi\sigma r \equiv \Delta r$, which implies a lower limit on the surface density of solids, viz.,

$$\frac{\pi\sigma_{\rm d}r^2}{M_\oplus} \gtrsim \frac{1}{2}\left(\frac{M}{M_\oplus}\right)\left(\frac{r\Omega}{c}\right)\left(\frac{\sigma_{\rm g}}{\sigma_{\rm d}}\right)^{1/4}. \tag{8}$$

For a 15 $M_\oplus$ planet in a disk with $c/r\Omega \sim O(10^{-1})$, Equation 8 reads $\sim 75(\sigma_{\rm g}/\sigma_{\rm d})^{1/4}$ Earth masses with a corresponding surface density of $\sigma_{\rm d} \gtrsim 0.7(\sigma_{\rm g}/\sigma_{\rm d})^{1/4}(r/30{\rm AU})^2{\rm g\ cm^{-2}}$. For the parameters adopted in Figure 1, over-damping may occur at less than $10M_\oplus$ as indicated by the black circle terminating the time line. A solar composition disk would require $\pi\sigma_{\rm d}r^2 \sim 200M_\oplus$, $\sigma_{\rm d} \sim 2$ g cm^{-2} to avoid this.

Concerning the second proviso, the orbital decay time of embryos $\tau_{\rm dw}^a$ due to type I decay is

$$\tau_{\rm dw}^a \sim \frac{c_{\rm a}^{-1}}{\Omega}\left(\frac{M_\star}{M}\right)\left(\frac{M_\star}{\pi\sigma_{\rm g}r^2}\right)\left(\frac{c}{r\Omega}\right)^2, \tag{9}$$

which exceeds the dispersion velocity damping time [Equation 4] by a factor of order $c_{\rm e}(r\Omega/c)^2/c_{\rm a}$, where c_a is the so-called torque asymmetry parameter (Goldreich & Tremaine, 1980; Ward 1986, 1989, 1997; Artymowicz, 1993). For

[1] In this case, further encounters must be generated via differential drift rate (Ward 1993; Tanaka & Ida 1999) which results in a different time scale.

our problem, this reads $\tau_{\rm dw}^{a} \approx 2.7 \times 10^8 c_{\rm a}^{-1} \sigma_{\rm d,cgs}^{-1} (M_{\oplus}/M)$ yr. Current best efforts at estimating the torque asymmetry parameter for a body in a circular orbit suggest $c_{\rm a} = O(1)$ (Tanaka, Takeuchi, & Ward 2001). Equation 9 is also shown in Figure 1 for comparison.

The ratio of accretion time to decay time is

$$\frac{\tau_{\rm acc}}{\tau_{\rm dw}^{a}} \sim \left(\frac{c_{\rm a}}{c_{\rm acc}}\right) \frac{\rho R^2 r}{M_\star} \sqrt{\sigma_{\rm g}/\sigma_{\rm d}} \approx \frac{1}{3}\left(\frac{c_{\rm a}}{c_{\rm acc}}\right)\left(\frac{M}{M_\oplus}\right)^{2/3} \sqrt{\sigma_{\rm g}/\sigma_{\rm d}}, \tag{10}$$

which is independent of scale height and surface densities, except for their ratio. The time ratio is thus $\sim 2.3(M/M_\oplus)^{2/3}$ for the conditions of Figure 1, implying that objects a few tenths of an Earth mass are in danger of decaying out of the disk before they can grow much more (e.g., Ward 1996, 1998; Tanaka & Ida 1999). However, this situation could be mitigated by disk dissipation as discussed next.

4. Remnant Disk

From Equation 7, the e-folding time to grow $10^{-1} M_\oplus$ bodies at $r \sim 30$ AU is few $\times 10^7$ yr. Since the lifetime of gas disks are typically estimated to be of this order, the ratio $\sigma_{\rm g}/\sigma_{\rm d}$ is likely to be significantly less than its starting value by then. A decreased gas density slows the orbital decay rate proportionally, but has a more modest effect on the accretion time through the dispersion velocity, which approaches the sound speed of the gas. Indeed, this is why the ratio [Equation 10] depends on $\sqrt{\sigma_{\rm g}/\sigma_{\rm d}}$. Note that although removing the gas entirely would revert the system to the case where $F_{\rm g}$ is nearly unity, only a slight residual of the gas disk (i.e., $\sigma_{\rm g}/\sigma_{\rm d} \sim O(1)$) is capable of keeping dispersion velocities near Mach one. In fact, the time scale ratio can be made to approach unity for any desired mass, M, by choosing $\sigma_{\rm g}/\sigma_{\rm d} \sim 9(c_{\rm acc}/c_a)^2 (M_\oplus/M)^{4/3}$; for $M \sim 17 M_\oplus$, this reads $\sim 0.2(c_{\rm acc}/c_{\rm a})^2$. Again, the small, but non-negligible, H/He content of Neptune seems consistent with a minor fraction of gas having remained in the vicinity during its formation.

However, there is another key issue that alleviates the situation, that is the recent demonstration by Papaloizou & Larwood (2000) that large eccentricities (i.e., $e \sim v/r\Omega$) also modify the migration rate by elevating the importance of terms in the embryos' disturbing functions that are higher order in e. The added resonant interactions with the disk result in an extra factor of order $\approx [1+(e\,r\,\Omega/c)^5]/[1-(e\,r\,\Omega/c)^4]$ in the migration time scale, $\tau_{\rm dw}^{a}$. From Equation 5, this results in an additional factor of $[1+(\sigma_{\rm d}/\sigma_{\rm g})^{5/4}]/[1-\sigma_{\rm d}/\sigma_{\rm g}]$ in Equation 9. Thus, the migration can slow and even reverse direction as the disk dissipates and the radial excursions of the embryo ($\delta r \sim e\,r \sim v/\Omega$) become comparable to the disk scale height, $h \sim c/\Omega$. Under such conditions, embryos may instead execute a more diffusive migrational behavior, as mutual scattering varies their dispersion velocities around the mean [Equation 5] between subsonic to mildly supersonic values. A strong scattering event tends to increase or decrease the dispersion velocity by amounts comparable to itself so that the PL-correction factor should vary between $\pm O(1)$ between scattering events. An embryo could

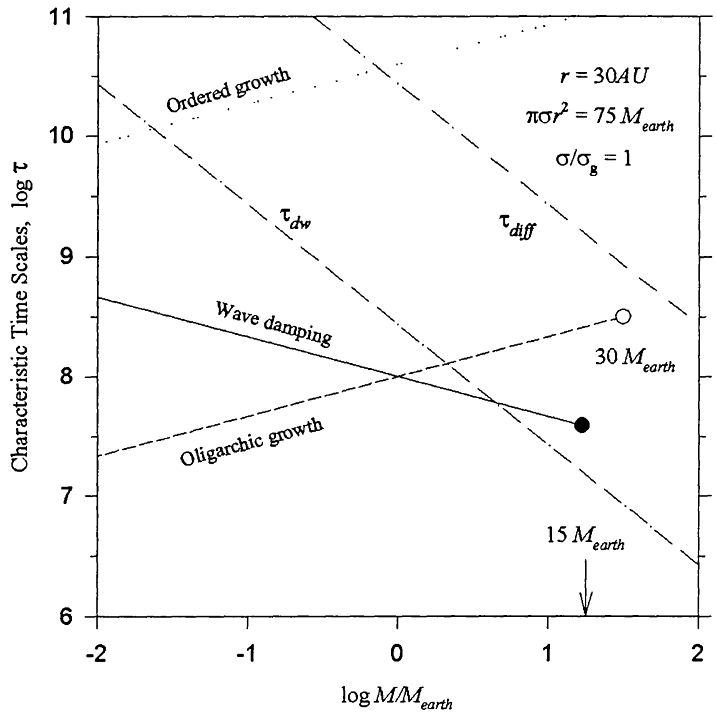

Figure 2. Same as Figure 1 except that gas to solid ratio is unity. Also shown is the inferred embryo diffusion time scale from incorporating the Papaloizou-Larwood correction factor into the orbit decay rate. For comparison, the orbit decay time without this factor is included as well.

drift a distance $\delta r \sim \pm r(\tau_{\rm scatt}/\tau^a_{\rm dw})$ during this time, where $\tau^a_{\rm dw}$ refers to the uncorrected migration rate of Equation 9. This implies a diffusion rate that satisfies $d(\Delta r)^2/dt \approx (\delta r)^2/\tau_{\rm scatt} \sim r^2 \tau_{\rm scatt}/(\tau^a_{\rm dw})^2$. Since equilibrium maintains $\tau_{\rm scatt} \sim \tau^e_{\rm dw}$, the time to diffuse a distance r is

$$\begin{aligned}\tau_{\rm diff} &\sim \tau_{\rm scatt}(r/\delta r)^2 \sim \tau_{\rm scatt}(\tau^a_{\rm dw}/\tau_{\rm scatt})^2 \\ &\approx \tau^a_{\rm dw}(\tau^a_{\rm dw}/\tau^e_{\rm dw}) \approx \tau^a_{\rm dw}(r\Omega/c)^2 \gg \tau^a_{\rm dw}. \end{aligned} \qquad (11)$$

With a gas density equal to that of solids, $\tau_{\rm acc} \sim 1\times 10^8 c_{\rm a} c_{\rm acc}^{-2} \sigma_{\rm d,cgs}^{-1} (M_\oplus/M)^{1/3}$ yr and $\tau_{\rm diff} \approx 2.7 \times 10^{10} c_{\rm a}^{-1}(M_\oplus/M)$ yr. Figure 2 shows these time scales as well as $\tau^a_{\rm dw}$ without the PL eccentricity correction. The critical mass to prevent over-damping is now $\sim 75\ M_\oplus$ so that the time line is extended to a Neptune mass.

For $M \sim 1\ M_\oplus$, the characteristic accretion time is $\sim 1\times 10^8$ yr, which is the most sluggish period of growth. Assuming an exponentially decaying gas density, $\sigma_{\rm g} \propto e^{-t/\tau_{\rm g}}$, a decrease by a factor of 0.02 within this time interval requires a disk dissipation e-folding time of $\tau_{\rm g} \sim 3\times 10^7$ yr. Note that recent direct observations of hydrogen suggests an extended the range of possible disk lifetimes around young stars to few $\times 10^7$ yr (Thi et al. 2001). Of course, $\tau_{\rm g} = \sigma_{\rm g}/\dot\sigma_{\rm g}$ itself may vary in time, depending on the source(s) of dissipation such as stellar UV and particle fluxes and/or disk viscosity.

5. Summary

We have described a set of circumstances that may help account for the ability of large, (i.e., $15M_{\oplus}$) planets to form at distances where their orbital velocities are smaller than the planetary escape velocities. We invoke the existence of a small remnant ($\sim 1\%$) of the original gas disk during the late-stage formation of Neptune—an assumption that seems in consonance with its small H/He content. The presence of the gas phase alters the dynamics of accreting embryos in two keys ways:

(1) Disk torques, primarily those generated at coorbiting Lindblad and vertical resonances, damp eccentricities and inclinations among a swarm of mutually interacting embryos. The resulting equilibrium dispersion velocities are comparable to the sound speed of the gas and are independent of embryo mass (Ward 1993). This promotes accretion by enhancing the collisional cross section by a factor $F_g \sim (v_{esc}/c)^2 \approx 3 \times 10^2 (M/M_{\oplus})^{2/3}$.

(2) The orbital decay of embryos due to disk torques is largely stalled. This occurs because coupling between the orbit and the disk is weaker for eccentric orbits due to the larger relative velocity between the protoplanet and local disk material (Papaloizou & Larwood 2000). When eccentricities are comparable to the scale height of the gas disk, $e \sim c/r\Omega$, terms in the disturbing function proportional to e^n, $n > 0$, begin to contribute significantly to the migration rate. The resulting net torque is considerably reduced, and can undergo a sign reversal. Instead of secular orbital decay, embryos may execute slow radial diffusion on a much longer time scale. These new insights into protoplanet-disk interactions suggest that an in situ formation of Neptune may be possible in a few $\times 10^8$ yr, and further exploration of accretion in the presence of a remnant gas disk seems warranted. These should include numerical experiments similar to those performed by Papaloizou and Larwood at 1 AU, but conducted for the outer Solar System with a time dependent gas phase. The results of such an investigation may not only illuminate the existence of Uranus and Neptune, but could also place constraints on models of nebula dissipation.

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Discussion

Martin Duncan: You talked about growing several of these embryos out to a tenth of an Earth mass or something and then going on. How sensitive is either the tidal migration inward or the damping time to the presence of other comparable-size embryos?

Bill Ward: Well, we really don't know that yet because detailed models are still to be developed, but it's an important consideration. However, you notice that the time scale for accretional growth assisted by wave damping has a downward slope in the figures, indicating that there's actually a runaway going on. The point is that you may well get one object that tends to out pace the others, so fully comparable embryos may not exist. They may be fairly large, but they're probably smaller than the dominant one, which would pretty much control the situation. There's not really an oligarchic growth at this point.