Proposal for PhD studentship -
Realisations of Graphs as Frameworks

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Graphs with geometrical constraints provide natural models for a variety of applications, including CAD, sensor networks and flexibility in molecules. Given a graph and prescribed lengths for its edges, a basic problem is to determine whether it has a straight line realisation in Euclidean d-dimensional space with these given lengths. We refer to such a realisation as a bar-and-joint framework. Given such a framework, one may also ask whether it is unique, either globally, or with respect to local movement (rigidity). The rigidity question has a strong mathematical pedigree, going back to a conjecture of Euler that every 3-dimensional polyhedron is rigid. The existence, global uniqueness, and rigidity problems are known to be NP-hard. However, this hardness relies on algebraic relations between coordinates of vertices, and for practical purposes it is natural to study generic realisations. Laman gave a combinatorial characterization for the rigidity of generic 2-dimensional frameworks in 1970, and Jackson and Jordán subsequently characterized their global rigidity in 2005. No combinatorial characterizations for rigidity or global rigidity are known in higher dimensions.

The project will consider related problems. One such problem, characterising graphs which admit a radically solvable realisation, is motivated by specific applications to CAD and would involve collaboration with J C Owen (Siemens, Park House, Cambridge).