Random Permutation Structures
A. Gnedin

Models for random permutations with nonuniform probability distribution are ubiquitous in many branches of pure and applied mathematics. Many models step away from the uniformity by ‘biasing’ the distribution of some permutation statistic, which may be either a numerical statistic (the number of cycles, descents, etc) or a more complex characteristic associated with the factoring into cycles or decomposition into increasing runs. A classical instance is the family Ewens’ distributions [4] under which the probability of permutation is proportional to $\theta^c$, where $c$ is the number of cycles and $\theta$ a positive parameter. The idea of permutation structure complements this approach to nonuniformity by also requiring consistency for various sizes $n$. From a viewpoint, a random permutation structure is a process in which permutation grows bit-by-bit as elements added, for instance like in the famous ‘Chinese Restaurant Process’, where elements are successively inserted in cycles using a simple rule [5].

A combinatorial part of the project aims to understand the compatibility of permutation statistics with systems of projections that connect various sizes, to see how the structures behave under transformations and what is the nature of the projective limit of the finite sets of permutations. Once the framework is established, a fundamental characterisation problem is representing a generic permutation structure in the spirit of de Finetti’s theorem, that is in the form of a convex mixture of some basic processes (see e.g. [3], [2]). The latter includes understanding the topology of the boundary (which itself may be a finite or infinite-dimensional simplex, or some kind of ‘discrete’ set) as well as the decomposability of the family of uniform distributions. Apart from the classification issues, the circle of questions of interest for applications is constructing Markovian permutation-valued processes with tractable transition probabilities, and analysing the asymptotic features of big permutations (see e.g. [1]).

References


