Designs for one-sided neighbour effects

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For Prem Narain on his 70th birthday

1 Introduction

In agriculture and allied subjects, the treatment applied to one experimental plot may affect the response on neighbouring plots as well as the response on the plot to which it is applied. In cereal crops or sunflowers, tall varieties may shade the plot on their North side [13, 21]. In pesticide or fungicide experiments, part of the treatment may spread to the plot immediately downwind; so may spores from untreated plots [11]. These are both examples of *one-sided* effects. In plants with an important root system, such as potatoes, varieties which germinate earlier will establish their roots and take nutrients from adjoining plots on both sides if the crop is grown in linear ridges [22], or on all sides if the crop is grown in a two-dimensional area with no gaps. Similar effects are reported on oil-seed rape [3], on field beans [14], in anti-feedants [20], in forestry [16] and in horticulture [8]. These effects are variously called *neighbour effects* or *competition effects* or *interference effects*.

This paper is concerned only with the first type of effect, the one-sided neighbour effect.

If the plots form a single long line, with plots numbered from 1 to n, assume that the neighbour effect is from plot j to plot $j+1$. Denote by $T(j)$ the treatment on plot j. With a one-sided neighbour effect, the response on plot j depends both on $T(j)$ and on $T(j-1)$. Some suitable designs for this situation have been given by Finney and Outhwaite [10], Dyke and Shelley [9] and Lewis [17].

However, it is more common to arrange the plots in separated linear blocks. Assume that there are b blocks of size k, and v treatments. Now denote by $T(i,j)$ the treatment on plot j of block i, and by Y_{ij} the response on that plot.

2 Models and effects

The simplest model for a one-sided neighbour effect is

$$\mathrm{E}(Y_{ij}) = \beta_i + \tau_{T(i,j)} + \alpha_{T(i,j-1)} \tag{1}$$

and

$$\mathrm{Cov}(\mathbf{Y}) = \sigma^2 \mathbf{I},$$

where $\beta_1, \ldots, \beta_b$ are (unknown) block effects, $\tau_1, \ldots, \tau_v$ are (unknown) *direct* effects of the treatments, $\alpha_1, \ldots, \alpha_v$ are (unknown) neighbour effects of the treatments, and σ^2 is the (unknown) variance per plot.

Some more complicated models for $\mathrm{E}(Y_{ij})$ have been proposed. One is that

$$\mathrm{E}(Y_{ij}) = \begin{cases} \beta_i + \tau_{T(i,j)} + \alpha_{T(i,j-1)} & \text{if } T(i,j) \neq T(i,j-1) \\ \beta_i + \tau_{T(i,j)} & \text{if } T(i,j) = T(i,j-1). \end{cases} \tag{2}$$

This is tantamount to saying that each treatment has no neighbour effect on itself. For example, it may be argued that tall sunflowers shade shorter varieties but not other sunflowers of the same height. However, photosynthesis occurs in all the leaves of a plant, so a plant growing next to another plant of the same variety can clearly make less use of the sun than a plant with no shading.

What the experimenter usually seeks to find is the overall effect of a treatment when it is grown throughout a field [5, 12]. If treatment x is applied to every plot in block i then, under model (1),

$$\mathrm{E}(Y_{ij}) = \beta_i + \phi_x,$$

where $\phi_x = \tau_x + \alpha_x$. We call ϕ_x the *total effect* of treatment x. I think that those who have proposed model (2) have confused τ_x with ϕ_x.

A further model [15, 21] which may confuse the direct and total effects is:

$$\mathrm{E}(Y_{ij}) = \begin{cases} \beta_i + \tau_{T(i,j)} + \alpha_{T(i,j-1)} & \text{if } T(i,j) \neq T(i,j-1) \\ \beta_i + \tau_{T(i,j)} + \gamma_{T(i,j)} & \text{if } T(i,j) = T(i,j-1). \end{cases} \tag{3}$$

In this case $\phi_x = \tau_x + \gamma_x$.

More complicated still is the model which allows for full interaction between a treatment and its neighbour [12, 22]:

$$\mathrm{E}(Y_{ij}) = \beta_i + \tau_{T(i,j)} + \alpha_{T(i,j-1)} + \delta_{T(i,j),T(i,j-1)}. \tag{4}$$

In this case $\phi_x = \tau_x + \alpha_x + \delta_{xx}$.

There is a large literature on designs for the estimation of direct effects τ. For example, Philippeau, Azaïs and Monod [19] recommend that, if model (1) is appropriate, then it is efficient to use a neighbour-balanced design (to be defined in Section 3) and analyse for the simple model with no neighbour effects. Kunert and Stufken [15] assume model (3) and recommend designs in which the γ parameters are not estimable.

However, the aim of the experiment is surely to estimate the total effects ϕ. If model (2) or (3) or (4) holds, then the α parameters (and δ parameters, if any) are of no interest but the γ parameters (if any) are important. In this situation the only sensible way to conduct the experiment is to apply treatments to large areas such as whole fields, with guard areas in between. This is likely to be much more expensive, have smaller true replication, and have larger variability than an experiment in smaller plots.

If model (1) holds then we can still conduct an experiment in small plots in linear blocks. There is a difficulty about plot 1 of each block, because there is apparently no neighbour effect to apply to it. However, we should really include a parameter α_0 for the effect of 'no neighbour'. Rather than fit this extra parameter, an alternative that is often recommended is to have a *border plot* before plot 1 of each block i. A treatment $T(i,0)$ is applied to this plot but its response is not measured. It is convenient if $T(i,0) = T(i,k)$, because then each neighbour effect occurs the same number of times in block j as its corresponding direct effect. A bordered block design with this property is called *circular*.

There is now a design dilemma. To estimate $\tau_x + \alpha_x$ well, we need many adjacent pairs of plots that both have treatment x. On the other hand, to allow for block effects efficiently, we do not want any treatment to occur more than once in any block, if $k \leq v$. However, adjacent plots are always in the same block if blocks are well separated.

Bailey and Druilhet [4] sought to resolve this dilemma by finding circular block designs which are optimal for estimation of the total effects ϕ. They showed that if no treatment is ever adjacent to itself then circular block designs which are binary (no treatment occurs more than once in a block), balanced (in the usual sense that every pair of distinct treatments is in the same number of blocks) and neighbour-balanced, are optimal for the estimation of total effects. Azaïs, Bailey and Monod [2] gave a table of such designs for $b = v$, $k = v - 1$ and $b = v - 1$, $k = v$, with instructions for their use.

Bailey and Druilhet also showed that if b is large then designs with self-neighbours are better than those without, if $k \geq 5$. They describe a class of optimal designs which can certainly be realised if $b = v!$, which is usually too large for practical use. The remainder of this paper gives optimal designs for the smallest possible value of b, for given small values of k and v.

3 Properties of the designs

Each design is *balanced* in the sense that there is an integer μ such that every pair of distinct treatments has concurrence μ. Here the *concurrence* of treatments x and y means the number of pairs of plots in the same block with one receiving treatment x and the other receiving treatment y. Each design is also *neighbour-balanced* in the sense that there is an integer λ such that every treatment is followed by each other treatment λ times.

Bailey and Druilhet [4] give the optimal number s of treatments to put in each block, for each block size k with $3 \leq k \leq 16$. Part of this information is reproduced in Table 1. When $k = 4$ there are three values of s, all optimal. Of the s treatments in any block, n_1 occur m times and n_2 occur $m+1$ times, where m is the integer part of k/s, $n_2 = k - sm$ and $n_1 = s - n_2$. Each block contributes $\theta/2$ to the sum of the concurrences, where

$$\begin{aligned}\theta &= n_1(n_1-1)m^2 + n_2(n_2-1)(m+1)^2 + 2n_1n_2m(m+1) \\ &= sm(m+1) + k(k-2m-1).\end{aligned}$$

Hence

$$b\theta = v(v-1)\mu. \tag{5}$$

Bailey and Druilhet [4] show that all occurrences of any one treatment in any one block must be in a single sequence of adjacent plots (possibly including both the last plot and the first plot), so each block contributes s to the sum of neighbour adjacencies. Hence

$$bs = v(v-1)\lambda. \tag{6}$$

k	3	4	4	4	5	6	7	8	9
s	3	2	3	4	3	3	4	4	4
m	1	2	1	1	1	2	1	2	2
n_1	3	2	2	4	1	3	1	4	3
n_2	0	0	1	0	2	0	3	0	1
θ	6	8	10	12	16	24	36	48	60

Table 1: When the blocks have size k, then s treatments should appear in each block, with n_1 appearing m times and n_2 appearing $m + 1$ times, so each block contributes θ to the sum of the concurrences

Note that if θ and s are coprime then b must be a multiple of $v(v-1)$.

4 The tables, and how to use them

Tables 3–6, supplemented by the text in Section 5, give the smallest designs with the properties in Section 3, for the range $3 \leq k \leq 9$ and $s \leq v \leq 10$. Apart from the exceptions mentioned in Section 5, each given design has the parameters which are the smallest solutions to Equations (5) and (6).

To use these, first choose one of the designs for the appropriate values of v and k. If $k = 4$ there may be a choice of design. In the tables, the blocks are shown as columns, to save space. Randomly allocate the columns of the chosen design to the actual blocks. In each block independently, randomly choose a number l between 1 and k inclusive, and move the treatment on plot i to plot $i + l$ modulo k. Finally, in each block, put the treatment on plot k onto the border plot before plot 1.

For example, if $v = k = 5$ then start with the design in Table 5(a), which has 20 blocks. Randomization can produce the layout in Table 2, where the blocks are shown as rows, with the border plot at the left-hand end.

5	1	1	2	2	5
2	2	4	4	5	2
3	1	2	2	3	3
5	5	1	3	3	5
2	3	3	4	4	2
1	1	2	4	4	1
4	4	5	5	3	4
3	1	1	5	3	3
5	4	4	3	3	5
1	4	4	2	2	1
5	5	4	4	1	5
5	5	3	3	2	5
1	5	5	2	1	1
3	1	1	4	4	3
5	2	2	3	5	5
4	2	2	5	5	4
1	1	4	5	5	1
3	4	1	1	3	3
2	2	1	1	3	2
3	3	2	2	4	3

Table 2: One layout obtained by randomizing the design in Table 5(a)

5 Tables of designs

5.1 Block size three

When $k = 3$ then $s = 3$ and the designs are just *Mendelsohn triple systems* [18]. As Colburn and Rosa [7] show, there is a design corresponding to the smallest integer solutions of Equations (5) and (6) except when $v = 6$. These are given in Table 3. The second smallest solution for $v = 6$ corresponds to the design in Table 3(d).

1	1
2	3
3	2

(a) $v = 3$, $b = 2$, $\mu = 2$, $\lambda = 1$

3	4	1	2
1	2	3	4
2	1	4	3

(b) $v = 4$, $b = 4$, $\mu = 2$, $\lambda = 1$

1	3	1	4	1	5	1	4	1	5	1	5	2	4	2	5	2	5	3	5
2	2	2	2	2	2	3	3	3	3	4	4	3	3	3	3	4	4	4	4
3	1	4	1	5	1	4	1	5	1	5	1	4	2	5	2	5	2	5	3

(c) $v = 5$, $b = 20$, $\mu = 6$, $\lambda = 3$

1	2	3	4	5	1	2	3	4	5	6	6	6	6	6	5	1	2	3	4
2	3	4	5	1	3	4	5	1	2	2	3	4	5	1	3	4	5	1	2
6	6	6	6	6	5	1	2	3	4	1	2	3	4	5	1	2	3	4	5

(d) $v = 6$, $b = 20$, $\mu = 4$, $\lambda = 2$

1	2	3	4	5	6	7	4	5	6	7	1	2	3
2	3	4	5	6	7	1	2	3	4	5	6	7	1
4	5	6	7	1	2	3	1	2	3	4	5	6	7

(e) $v = 7$, $b = 14$, $\mu = 2$, $\lambda = 1$

1	2	3	4	5	6	7	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
4	5	6	7	1	2	3	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6
2	3	4	5	6	7	1	1	2	3	4	5	6	7	2	3	4	5	6	7	1	4	5	6	7	1	2	3

1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
3	4	5	6	7	1	2	2	3	4	5	6	7	1	6	7	1	2	3	4	5	5	6	7	1	2	3	4
5	6	7	1	2	3	4	3	4	5	6	7	1	2	2	3	4	5	6	7	1	2	3	4	5	6	7	1

(f) $v = 8$, $b = 56$, $\mu = 6$, $\lambda = 3$

1	3	4	6	7	9	1	7	2	8	3	9	1	9	2	7	3	8	1	8	2	9	3	7
2	2	5	5	8	8	4	4	5	5	6	6	5	5	6	6	4	4	6	6	4	4	5	5
3	1	6	4	9	7	7	1	8	2	9	3	9	1	7	2	8	3	8	1	9	2	7	3

(g) $v = 9$, $b = 24$, $\mu = 2$, $\lambda = 1$

1	4	7	0	0	0	0	0	0	0	0	0	1	7	2	8	3	9	1	9	2	7	3	8	1	8	2	9	3	7
2	5	8	5	6	4	2	3	1	8	9	7	4	4	5	5	6	6	5	5	6	6	4	4	6	6	4	4	5	5
3	6	9	4	5	6	1	2	3	7	8	9	7	1	8	2	9	3	9	1	7	2	8	3	8	1	9	2	7	3

(h) $v = 10$, $b = 30$, $\mu = 2$, $\lambda = 1$

Table 3: Designs for blocks of size 3 ($k = 3$, $s = 3$, $\theta = 6$)

5.2 Block size four

When $k = 4$ then s may be 2 or 3 or 4. If $s = 2$ then each block has the cyclic pattern (x, x, y, y). Using one such block for each unordered pair $\{x, y\}$ of treatments gives a design with $b = v(v-1)/2$. The designs in parts (a), (c), (f), (h), (j) and (l) of Table 4 have this form. If $s = 3$ then Equations (5) and (6) give $10b = v(v-1)\mu$ and $3b = v(v-1)\lambda$, whose smallest integer solution has $b = v(v-1)$, so these designs are no improvement on those with $s = 2$ and therefore none are shown in Table 4.

When $s = 4$ the designs are known as *oriented* balanced incomplete-block designs or *perfect Mendelsohn designs* [18], and are related to *directed Whist tournaments* [1]. Now Equations (5) and (6) give $\mu = 3\lambda$ and $4b = v(v-1)\lambda$. Table 4 includes designs for the smallest integer solutions to these equations except for $v = 4$ (when trial and error quickly shows that there is no solution with $b = 3$), and $v = 8$ (where [6] shows that there is no solution with $b = 14$).

1	1	2
1	1	2
2	3	3
2	3	3

(a) $v = 3,\ s = 2,\ b = 3,\ \theta = 8,\ \mu = 4,\ \lambda = 1$

1	1	1	2	2	2
2	3	4	1	4	4
3	4	2	4	3	1
4	2	3	3	1	3

(b) $v = 4,\ s = 4,\ b = 6,\ \theta = 12,\ \mu = 6,\ \lambda = 2$

1	1	1	2	2	3
1	1	1	2	2	3
2	3	4	3	4	4
2	3	4	3	4	4

(c) $v = 4,\ s = 2,\ b = 6,\ \theta = 8,\ \mu = 4,\ \lambda = 1$

1	2	3	4	5
2	3	4	5	1
4	5	1	2	3
3	4	5	1	2

(d) $v = 5,\ s = 4,\ b = 5,\ \theta = 12,\ \mu = 3,\ \lambda = 1$

6	6	6	6	6	6	6	6	6	6	2	2	2	2	3
1	1	2	3	5	2	3	4	4	5	4	4	5	1	5
2	3	1	1	4	5	4	3	5	2	5	3	1	5	4
3	4	4	5	1	1	2	5	2	3	3	1	4	3	1

(e) $v = 6,\ s = 4,\ b = 15,\ \theta = 12,\ \mu = 6,\ \lambda = 2$

1	1	1	1	1	2	2	2	2	3	3	3	4	4	5
1	1	1	1	1	2	2	2	2	3	3	3	4	4	5
2	3	4	5	6	3	4	5	6	4	5	6	5	6	6
2	3	4	5	6	3	4	5	6	4	5	6	5	6	6

(f) $v = 6,\ s = 2,\ b = 15,\ \theta = 8,\ \mu = 4,\ \lambda = 1$

7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6
1	2	3	4	5	6	7	1	2	3	4	5	6	7	2	3	4	5	6	7	1
3	4	5	6	7	1	2	4	5	6	7	1	2	3	6	7	1	2	3	4	5
2	3	4	5	6	7	1	3	4	5	6	7	1	2	4	5	6	7	1	2	3

(g) $v = 7,\ s = 4,\ b = 21,\ \theta = 12,\ \mu = 6,\ \lambda = 2$

1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	4	4	4	5	5	6
1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	4	4	4	5	5	6
2	3	4	5	6	7	3	4	5	6	7	4	5	6	7	5	6	7	6	7	7
2	3	4	5	6	7	3	4	5	6	7	4	5	6	7	5	6	7	6	7	7

(h) $v = 7,\ s = 2,\ b = 21,\ \theta = 8,\ \mu = 4,\ \lambda = 1$

Table 4: Designs for blocks of size four ($k = 4$)

7 1 2 3 4 5 6 8 8 8 8 8 8 8 4 5 6 7 1 2 3 2 3 4 5 6 7 1
1 2 3 4 5 6 7 1 2 3 4 5 6 7 2 3 4 5 6 7 1 6 7 1 2 3 4 5
2 3 4 5 6 7 1 6 7 1 2 3 4 5 1 2 3 4 5 6 7 1 2 3 4 5 6 7
4 5 6 7 1 2 3 2 3 4 5 6 7 1 7 1 2 3 4 5 6 8 8 8 8 8 8 8

(i) $v = 8$, $s = 4$, $b = 28$, $\theta = 12$, $\mu = 6$, $\lambda = 2$

1 1 1 1 1 1 1 2 2 2 2 2 2 3 3 3 3 3 4 4 4 4 5 5 5 6 6 7
1 1 1 1 1 1 1 2 2 2 2 2 2 3 3 3 3 3 4 4 4 4 5 5 5 6 6 7
2 3 4 5 6 7 8 3 4 5 6 7 8 4 5 6 7 8 5 6 7 8 6 7 8 7 8 8
2 3 4 5 6 7 8 3 4 5 6 7 8 4 5 6 7 8 5 6 7 8 6 7 8 7 8 8

(j) $v = 8$, $s = 2$, $b = 28$, $\theta = 8$, $\mu = 4$, $\lambda = 1$

5 6 4 2 3 1 8 9 7 2 3 1 8 9 7 5 6 4
6 4 5 3 1 2 9 7 8 4 5 6 1 2 3 7 8 9
9 7 8 6 4 5 3 1 2 3 1 2 9 7 8 6 4 5
8 9 7 5 6 4 2 3 1 7 8 9 4 5 6 1 2 3

(k) $v = 9$, $s = 4$, $b = 18$, $\theta = 12$, $\mu = 3$, $\lambda = 1$

1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 4 4 4 4 4 4
1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 4 4 4 4 4 4
2 3 4 5 6 7 8 9 0 3 4 5 6 7 8 9 0 4 5 6 7 8 9 0 5 6 7 8 9 0
2 3 4 5 6 7 8 9 0 3 4 5 6 7 8 9 0 4 5 6 7 8 9 0 5 6 7 8 9 0

5 5 5 5 5 6 6 6 6 7 7 7 8 8 9
5 5 5 5 5 6 6 6 6 7 7 7 8 8 9
6 7 8 9 0 7 8 9 0 8 9 0 9 0 0
6 7 8 9 0 7 8 9 0 8 9 0 9 0 0

(l) $v = 10$, $s = 2$, $b = 45$, $\theta = 8$, $\mu = 4$, $\lambda = 1$

1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9
2 3 4 5 6 7 8 9 1 3 4 5 6 7 8 9 1 2 4 5 6 7 8 9 1 2 3
6 7 8 9 1 2 3 4 5 2 3 4 5 6 7 8 9 1 9 1 2 3 4 5 6 7 8
4 5 6 7 8 9 1 2 3 7 8 9 1 2 3 4 5 6 6 7 8 9 1 2 3 4 5

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9
2 3 4 5 6 7 8 9 1 9 1 2 3 4 5 6 7 8
4 5 6 7 8 9 1 2 3 7 8 9 1 2 3 4 5 6

(m) $v = 10$, $s = 4$, $b = 45$, $\theta = 12$, $\mu = 6$, $\lambda = 2$

Table 4: (continued) Designs for blocks of size four ($k = 4$)

5.3 Block size five

When $k = 5$ then $s = 3$, so Equations (5) and (6) give $16b = v(v-1)\mu$ and $3b = v(v-1)\lambda$, so b must be a multiple of $v(v-1)$. If $v = 3$, 4, 7, 9 or 10 then Table 3 gives a design with $s = k = 3$ and $b = v(v-1)/3$. Replace each block of the form (x, y, z) by the three blocks (x, y, y, z, z), (x, x, y, z, z) and (x, x, y, y, z). Designs for $v = 5$, 6 and 8 with $b = v(v-1)$ are given in Table 5.

1	2	3	4	5	2	3	4	5	1	3	4	5	1	2	4	5	1	2	3
2	3	4	5	1	4	5	1	2	3	1	2	3	4	5	3	4	5	1	2
2	3	4	5	1	4	5	1	2	3	1	2	3	4	5	3	4	5	1	2
3	4	5	1	2	1	2	3	4	5	4	5	1	2	3	2	3	4	5	1
3	4	5	1	2	1	2	3	4	5	4	5	1	2	3	2	3	4	5	1

(a) $v = 5$, $b = 20$, $\mu = 16$, $\lambda = 3$

6	6	6	6	6	1	2	3	4	5	2	3	4	5	1	5	1	2	3	4	4	5	1	2	3	4	5	1	2	3
2	3	4	5	1	3	4	5	1	2	5	1	2	3	4	6	6	6	6	6	3	4	5	1	2	5	1	2	3	4
2	3	4	5	1	3	4	5	1	2	5	1	2	3	4	6	6	6	6	6	3	4	5	1	2	5	1	2	3	4
1	2	3	4	5	4	5	1	2	3	6	6	6	6	6	2	3	4	5	1	5	1	2	3	4	3	4	5	1	2
1	2	3	4	5	4	5	1	2	3	6	6	6	6	6	2	3	4	5	1	5	1	2	3	4	3	4	5	1	2

(b) $v = 6$, $b = 30$, $\mu = 16$, $\lambda = 3$

2	3	4	5	6	7	1	1	2	3	4	5	6	7	8	8	8	8	8	8	8	7	1	2	3	4	5	6
1	2	3	4	5	6	7	8	8	8	8	8	8	8	7	1	2	3	4	5	6	4	5	6	7	1	2	3
1	2	3	4	5	6	7	8	8	8	8	8	8	8	7	1	2	3	4	5	6	4	5	6	7	1	2	3
4	5	6	7	1	2	3	7	1	2	3	4	5	6	2	3	4	5	6	7	1	8	8	8	8	8	8	8
4	5	6	7	1	2	3	7	1	2	3	4	5	6	2	3	4	5	6	7	1	8	8	8	8	8	8	8

3	4	5	6	7	1	2	2	3	4	5	6	7	1	6	7	1	2	3	4	5	5	6	7	1	2	3	4
5	6	7	1	2	3	4	3	4	5	6	7	1	2	2	3	4	5	6	7	1	2	3	4	5	6	7	1
5	6	7	1	2	3	4	3	4	5	6	7	1	2	2	3	4	5	6	7	1	2	3	4	5	6	7	1
1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7

(c) $v = 8$, $b = 56$, $\mu = 16$, $\lambda = 3$

Table 5: Designs for blocks of size five ($k = 4$, $s = 3$, $\theta = 16$): see text for other numbers of treatments

5.4 Block size six

When $k = 6$ then $s = 3$ and every block has the form (x, x, y, y, z, z). Use the design from Table 3 for the appropriate value of v, and double the occurrences of each entry. For example, if $v = 7$ then the first block is $(1, 1, 2, 2, 4, 4)$. Note that, after randomization, there are six possibilities for this block, including

2	4	4	1	1	2	2

and

4	4	1	1	2	2	4

.

5.5 Block size seven

When $k = 7$ then $s = 4$, so Equations (5) and (6) give $\mu = 9\lambda$ and $4b = v(v - 1)\lambda$. Also, every within-block contribution to concurrence is either 4 or 2, so μ is even and hence λ is even. Each block has a single unrepeated treatment, so, to maintain symmetry, b must be a multiple of v. Table 6 shows the smallest design for $v = 4$, 6, 7, 8 and 9.

When $v = 5$, use the design in Table 4 with $s = 4$ and replace each block of the form (w, x, y, z) by the four blocks (w, x, x, y, y, z, z), (w, w, x, y, y, z, z), (w, w, x, x, y, z, z) and (w, w, x, x, y, y, z).

When $v = 10$ the smallest solution to the equations has $b = 90$. A design with 90 blocks of 7 plots each is probably too large for practical purposes, so no design is tabulated.

5.6 Block size eight

When $k = 8$ then $s = 4$ and every block has the form (w, w, x, x, y, y, z, z). Use the design from Table 4 for the appropriate value of v with $s = 4$, and double the occurrence of each entry.

5.7 Block size nine

When $k = 9$ then $s = 4$ and again we find that $4b = v(v - 1)\lambda$, λ is even and v divides b. Use the designs for $k = 7$ and replace each block of the form (w, x, x, y, y, z, z) by the block $(w, w, w, x, x, y, y, z, z)$.

1	2	3	4	2	3	3	1	1	2	4	4
2	1	4	3	3	1	2	3	4	4	1	2
2	1	4	3	3	1	2	3	4	4	1	2
3	4	1	2	1	2	4	4	2	3	3	1
3	4	1	2	1	2	4	4	2	3	3	1
4	3	2	1	4	4	1	2	3	1	2	3
4	3	2	1	4	4	1	2	3	1	2	3

(a) $v = 4$, $b = 12$, $\mu = 36$, $\lambda = 4$

6	4	4	5	2	3	6	3	2	5	5	6	1	5	3	4	1	3	1	6	1	2	2	4	2	6	3	5	1	4
4	3	5	2	3	6	3	4	4	3	4	5	5	6	4	6	3	5	6	2	4	1	5	2	3	1	2	3	2	3
4	3	5	2	3	6	3	4	4	3	4	5	5	6	4	6	3	5	6	2	4	1	5	2	3	1	2	3	2	3
3	5	2	6	6	5	4	2	5	2	1	4	6	3	6	1	5	4	2	5	6	4	1	5	6	2	1	2	4	1
3	5	2	6	6	5	4	2	5	2	1	4	6	3	6	1	5	4	2	5	6	4	1	5	6	2	1	2	4	1
5	6	6	4	5	2	2	6	3	4	6	1	3	1	1	3	4	1	5	1	2	6	4	1	1	3	5	1	3	2
5	6	6	4	5	2	2	6	3	4	6	1	3	1	1	3	4	1	5	1	2	6	4	1	1	3	5	1	3	2

(b) $v = 6$, $b = 30$, $\mu = 36$, $\lambda = 4$

7	1	2	3	4	5	6	1	2	3	4	5	6	7	7	1	2	3	4	5	6
1	2	3	4	5	6	7	4	5	6	7	1	2	3	2	3	4	5	6	7	1
1	2	3	4	5	6	7	4	5	6	7	1	2	3	2	3	4	5	6	7	1
3	4	5	6	7	1	2	3	4	5	6	7	1	2	6	7	1	2	3	4	5
3	4	5	6	7	1	2	3	4	5	6	7	1	2	6	7	1	2	3	4	5
2	3	4	5	6	7	1	7	1	2	3	4	5	6	4	5	6	7	1	2	3
2	3	4	5	6	7	1	7	1	2	3	4	5	6	4	5	6	7	1	2	3

(c) $v = 7$, $b = 21$, $\mu = 18$, $\lambda = 2$

Table 6: Designs for blocks of size seven ($k = 7$, $s = 4$, $\theta = 36$): see text for other numbers of treatments

7	1	2	3	4	5	6	1	2	3	4	5	6	7	2	3	4	5	6	7	1	4	5	6	7	1	2	3
1	2	3	4	5	6	7	2	3	4	5	6	7	1	1	2	3	4	5	6	7	2	3	4	5	6	7	1
1	2	3	4	5	6	7	2	3	4	5	6	7	1	1	2	3	4	5	6	7	2	3	4	5	6	7	1
2	3	4	5	6	7	1	4	5	6	7	1	2	3	7	1	2	3	4	5	6	1	2	3	4	5	6	7
2	3	4	5	6	7	1	4	5	6	7	1	2	3	7	1	2	3	4	5	6	1	2	3	4	5	6	7
4	5	6	7	1	2	3	7	1	2	3	4	5	6	4	5	6	7	1	2	3	7	1	2	3	4	5	6
4	5	6	7	1	2	3	7	1	2	3	4	5	6	4	5	6	7	1	2	3	7	1	2	3	4	5	6

8	8	8	8	8	8	8	1	2	3	4	5	6	7	6	7	1	2	3	4	5	2	3	4	5	6	7	1
1	2	3	4	5	6	7	6	7	1	2	3	4	5	1	2	3	4	5	6	7	6	7	1	2	3	4	5
1	2	3	4	5	6	7	6	7	1	2	3	4	5	1	2	3	4	5	6	7	6	7	1	2	3	4	5
6	7	1	2	3	4	5	2	3	4	5	6	7	1	8	8	8	8	8	8	8	1	2	3	4	5	6	7
6	7	1	2	3	4	5	2	3	4	5	6	7	1	8	8	8	8	8	8	8	1	2	3	4	5	6	7
2	3	4	5	6	7	1	8	8	8	8	8	8	8	2	3	4	5	6	7	1	8	8	8	8	8	8	8
2	3	4	5	6	7	1	8	8	8	8	8	8	8	2	3	4	5	6	7	1	8	8	8	8	8	8	8

(d) $v = 8$, $b = 56$, $\mu = 36$, $\lambda = 4$

1	2	3	7	8	9	4	5	6	4	5	6	1	2	3	7	8	9	1	2	3	7	8	9	4	5	6	6	4	5	3	1	2	9	7	8
2	3	1	8	9	7	5	6	4	1	2	3	7	8	9	4	5	6	6	4	5	3	1	2	9	7	8	2	3	1	8	9	7	5	6	4
2	3	1	8	9	7	5	6	4	1	2	3	7	8	9	4	5	6	6	4	5	3	1	2	9	7	8	2	3	1	8	9	7	5	6	4
5	6	4	2	3	1	8	9	7	2	3	1	8	9	7	5	6	4	2	3	1	8	9	7	5	6	4	9	7	8	6	4	5	3	1	2
5	6	4	2	3	1	8	9	7	2	3	1	8	9	7	5	6	4	2	3	1	8	9	7	5	6	4	9	7	8	6	4	5	3	1	2
4	5	6	1	2	3	7	8	9	5	6	4	2	3	1	8	9	7	9	7	8	6	4	5	3	1	2	1	2	3	7	8	9	4	5	6
4	5	6	1	2	3	7	8	9	5	6	4	2	3	1	8	9	7	9	7	8	6	4	5	3	1	2	1	2	3	7	8	9	4	5	6

(e) $v = 9$, $b = 36$, $\mu = 18$, $\lambda = 2$

Table 6: (continued) Designs for blocks of size seven ($k = 7$, $s = 4$, $\theta = 36$): see text for other numbers of treatments

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