Self-stabilization of chaotic domain oscillations in superlattices by time-delayed feedback control

J. Schlesner[1], A. Amann[1], N. Janson[1,2], W. Just[1,3], and E. Schöll[1]

[1]Institut für Theoretische Physik, TU Berlin, D-10623 Berlin, Germany
[2]Department of Mathematical Sciences, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK
[3]Theoretische Physik I, TU Chemnitz, 09107 Chemnitz, Germany

E-mail: `schoell@physik.tu-berlin.de`

Abstract. Semiconductor superlattices driven into the regime of negative differential conductivity often exhibit complex or even chaotic self-sustained current oscillations due to traveling field domains. For a reliable operation of a superlattice as an electronic oscillator such unpredictable and irregular conditions should be avoided. In this work we present a theoretical analysis, proposing a novel scheme to stabilize high-frequency domain oscillations in semiconductor superlattices by a time–delayed feedback loop.

PACS numbers: 72.20.Ht,73.61.-r,05.45.Gg,05.45.Pq

1. Introduction

Semiconductor superlattices [1] are well known to give rise to self-sustained current oscillations ranging from several hundred MHz [2, 3] to 150 GHz at room temperature [4] due to propagating high field domains. Such a highly nonlinear system can exhibit chaotic scenarios [5] even under dc conditions [6]. However, for a reliable operation of a superlattice as an electronic oscillator such unpredictable and irregular conditions should be avoided, which might not be easy in practice.

Here we propose a simple self-stabilizing scheme which should be easy to implement in practical devices. It uses a profound concept from nonlinear dynamics and chaos theory to control chaotic behavior, and stabilize intrinsically unstable time-periodic motion by a simple feedback loop with a time delay [7, 8, 9].

2. Model

Our model of a superlattice is based on sequential tunneling of electrons [10]. The resulting tunneling current density $J_{m\to m+1}(F_m, n_m, n_{m+1})$ from well m to well $m+1$ depends only on the electric field F_m between both wells and the electron densities n_m and n_{m+1} in the respective wells (in units of cm^{-2}). The electron densities are determined by the continuity equation

$$e\frac{dn_m}{dt} = J_{m-1\to m} - J_{m\to m+1} \quad \text{for } m = 1, \ldots N \tag{1}$$

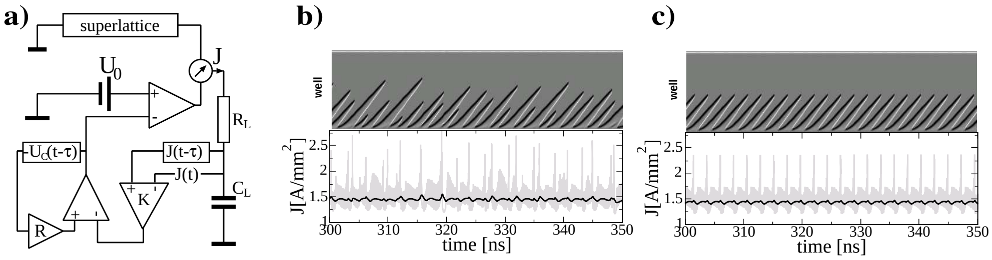

Figure 1. Self-stabilization of chaotic front dynamics by time-delayed feedback control. a) Control circuit including the low-pass filter (resistor R_L, capacitor C_L) with cut-off frequency $\alpha = 1/(R_L C_L)$ and the time-delayed feedback loop (K) and its extension to multiple time delays (R). b) Space-time plot of the uncontrolled charge density, and current density J vs. time. c) Same with global voltage control. (sequential tunneling model with $N = 100$ periods at $T = 20$K, $Al_{0.3}Ga_{0.7}As$ barriers of width $b = 5$nm and GaAs quantum wells of width $w = 8$nm, $N_D = 1.0 \times 10^{11}\mathrm{cm}^{-2}$, $\sigma = 0.5 \quad \Omega^{-1}\mathrm{m}^{-1}$, $U = 1.15$ V, $\tau = 2.29$ ns, $K = 3 \times 10^{-6} Vmm^2/A$, $R = 0.2$, $\alpha = 10^9 s^{-1}$.)

and the discrete version of Gauss's law $\epsilon_r \epsilon_0 (F_m - F_{m-1}) = -e(n_m - N_D)$, where N is the number of wells in the superlattice, ϵ_r and ϵ_0 are the relative and absolute permittivities, $e > 0$ is the electron charge, N_D is the donor density, and F_0 and F_N are the fields at the emitter and collector barrier, respectively. The total applied voltage U between emitter and collector imposes a global constraint $U = -\sum_{m=0}^{N} F_m d$, where d is the superlattice period. This, together with Poisson's equation, allows us to eliminate the field variables $F_m(n_1, ..., n_N, U)$ from the dynamic equations. At the contacts Ohmic boundary conditions $J_{0\to 1} = \sigma F_0$, $J_{N \to N+1} = \sigma F_N n_N / N_D$ are chosen, where σ is the Ohmic contact conductivity. If it is chosen appropriately, electron accumulation and depletion fronts are generated at the emitter. Those fronts form a traveling high field domain, with leading electron depletion front and trailing accumulation front. This leads to self-generated current oscillations. If the accumulation and depletion fronts have different velocities, they may collide in pairs and annihilate. At certain combinations of contact conductivity σ and voltage U, chaotic motion arises, when the annihilation of fronts of opposite polarity occurs at irregular positions within the superlattice, leading to complex bifurcation diagrams.

3. Results

We shall now introduce a time delayed feedback loop to control the chaotic front motion and stabilize a periodic oscillation mode which is inherent in the chaotic attractor (Fig. 1a). As a global output signal that is coupled back in the feedback loop, it is natural to use the total current density J. For the uncontrolled chaotic oscillations, the space-time plot of the electron densities and the current density J are given in Fig. 1b, showing irregular spikes $J(t)$ at those times when two fronts annihilate. Note that the grey current time trace is modulated by fast small-amplitude oscillations (due to well-to-well hopping of depletion and accumulation fronts in our discrete model) which are not resolved in the plot. However, as the variable J is fed back to the system for the purposes of control, these high–frequency oscillations render the control loop unstable. They need to be filtered out by using, e.g., the following low–pass filter:

$$\overline{J}(t) = \alpha \int_0^t J(t')e^{-\alpha(t-t')}\mathrm{d}t', \tag{2}$$

with a cut-off frequency α. The multiple differences of the filtered signal $\overline{J}$ (Fig. 1b and c, black curve) at time t and at delayed times $t-\tau$ are then used to modulate the voltage U across the superlattice:

$$U = U_0 + U_c(t), \quad U_c(t) = -K\left(\overline{J}(t) - \overline{J}(t-\tau)\right) + RU_c(t-\tau) \tag{3}$$

where U_0 is a time–independent external bias, and U_c is the control voltage. K is the amplitude of the control force, τ is the delay time, and R is a memory parameter.

Such a global control scheme is easy to implement experimentally. It is non-invasive in the sense that the control force vanishes when the target state of period τ has been reached. This target state is an unstable periodic orbit of the uncontrolled system. The period τ can be determined by observing the resonance-like behavior of the mean control force versus τ. The result of the control is shown in Fig. 1c. The front dynamics exhibits annihilation of front pairs at fixed positions within the superlattice, and stable periodic oscillations of the current are obtained. In Fig. 2 the Poincare sections for the uncontrolled and the controlled case are shown, clearly demonstrating the efficiency of control.

In conclusion, we have shown that time-delay autosynchronization by a feedback loop represents a convenient and simple scheme for the self-stabilization of high-frequency current oscillations associated with space–time patterns in semiconductor superlattices. This work was supported by DFG in the framework of Sfb 555.

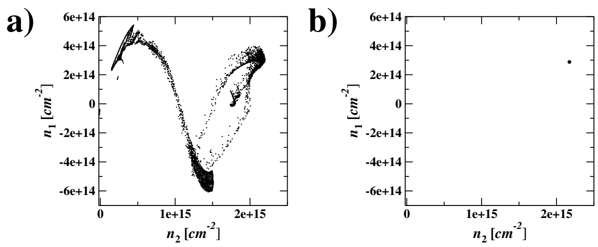

Figure 2. Poincare sections projected onto the (n_1, n_2) plane: a) without control (chaotic), b) with control (periodic). Parameters as in Fig. 1

[1] L. Esaki and R. Tsu, IBM J. Res. Develop. **14**, 61 (1970).
[2] J. Kastrup, R. Klann, H. T. Grahn, K. Ploog, L. L. Bonilla, J. Galán, M. Kindelan, M. Moscoso, and R. Merlin, Phys. Rev. B **52**, 13761 (1995).
[3] K. Hofbeck, J. Grenzer, E. Schomburg, A. A. Ignatov, K. F. Renk, D. G. Pavel'ev, Y. Koschurinov, B. Melzer, S. Ivanov, S. Schaposchnikov, and P. S. Kop'ev, Phys. Lett. A **218**, 349 (1996).
[4] E. Schomburg, R. Scheuerer, S. Brandl, K. F. Renk, D. G. Pavel'ev, Y. Koschurinov, V. Ustinov, A. Zhukov, A. Kovsh, and P. S. Kop'ev, Electronics Letters **35**, 1491 (1999).
[5] Y. Zhang, J. Kastrup, R. Klann, K. H. Ploog, and H. T. Grahn, Phys. Rev. Lett. **77**, 3001 (1996).
[6] A. Amann, J. Schlesner, A. Wacker, and E. Schöll, Phys. Rev. B **65**, 193313 (2002).
[7] K. Pyragas, Phys. Lett. A **170**, 421 (1992).
[8] J. E. S. Socolar, D. W. Sukow, and D. J. Gauthier, Phys. Rev. E **50**, 3245 (1994).
[9] E. Schöll, *Nonlinear spatio-temporal dynamics and chaos in semiconductors* (Cambridge University Press, Cambridge, 2001).
[10] A. Wacker, Phys. Rep. **357**, 1 (2002).