MAS115 Calculus I 2007-2008

Learning Outcomes

On completion of this course students will be expected to

1. know and use elements of set theory notation in the context of real line;
2. be able to solve algebraic equations and inequalities involving the square root and modulus function, e.g.
$$|x-3|+|2x-4|<10, \quad 2\sqrt{x-4}\le x+1, \quad \frac{|2x-4|}{x}<x+2;$$
3. understand the difference between equations and identities, and be able to prove simple identities and inequlities, e.g. $a^2+b^2\ge 2|ab|$;
4. know addition and double-angle formulas for trigonometric functions and use them to express values of trigonometric functions in the surds form, e.g. $\cos(\pi/12)=(1+\sqrt{3})/(2\sqrt{2})$ and $\cos^2(\pi/8)=(2+\sqrt{2})/4$;
5. be able to recognize odd, even, periodic, increasing, decreasing functions, e.g. is $f(x)=\sin(x^3)/\cos(x+\pi/2)$ odd, even or neither?;
6. understand the operation of composition of functions and the concept of functional inverse;
7. to able to recognize linear, quadratic, power, polynomial, algebraic, rational, trigonometric, exponential, hyperbolic and logarithmic functions and sketch their graphs; given the graph of $f(x)$ sketch the graph of $|f(ax+b)|$ or $af(|x|)+b$.
8. be able to manipulate piece-wise defined functions;
9. be able to calculate limits by substitution and by eliminating zero denominators, e.g.
$$\lim_{x\to -5}\frac{x-\sqrt{x+6}}{3-x}, \quad \lim_{x\to 3}\frac{x-\sqrt{x+6}}{3-x}, \quad \lim_{x\to 1}\frac{x^3-1}{x-1};$$
10. be able to calculate limits at infinity of rational functions and rational algebraic, e.g.
$$\lim_{x\to+\infty}\frac{100x^5+1}{x^{10}+4}, \quad \lim_{x\to+\infty}\frac{\sqrt{2x+1}-\sqrt{x}}{\sqrt{x}};$$
11. be able to calculate limits in indeterminate forms by a repeated use of l'Hopital rule, including limits involving $\frac{\sin x}{x}$ and $(1+\frac{1}{x})^x$, e.g.
$$\lim_{x\to 0}\frac{1-\cos^2(2x)}{\sin(x^2)}, \quad \lim_{x\to+\infty}x^2e^{-x}, \quad \lim_{x\to 0}(1+\sin^2 x)^{1/\sin(x^2)};$$
12. understand the concepts of rate of change and instantaneous rate of change;
13. know derivatives of power, trigonometric, exponential, hyperbolic, logarithmic and inverse trigonometric functions; know the basic rules of differentiation and use them to find derivatives of products and quotients;
14. know the chain rule and use it to find derivatives of composite functions, e.g.
$$\cos(1+e^{-x^2}), \quad \frac{\ln(1+\sin^2 x)}{\sin(x^2)};$$
15. be able to use derivatives to find intervals on which the given function is increasing or decreasing, find maxima and minima of functions;
16. be able to find tangents and normals to graphs of functions given in explicit, implicit and parametric forms;
17. be able to estimate change with differentials;

18. be able to sketch graphs of rational functions including finding asymptotes;
19. understand the concept of indefinite integral as anti-derivative;
20. know standard indefinite integrals and basic rules of indefinite integration;
21. be able to evaluate integrals by substitution with and without suitable hints, e.g.

$$\int x\sqrt{2x^2+1}dx, \quad \int \frac{dx}{\sqrt{4+x^2}}, \quad \int \sqrt{4-x^2}dx, \quad \int \frac{xdx}{\sqrt{x^2-3x+2}};$$

22. be able to evaluate integrals of rational functions by partial fractions;
23. be able to evaluate integrals by a repeated use of integration by parts, e.g.

$$\int x^2 \ln x dx, \quad \int e^{2x} \sin x dx;$$

24. understand the concept of definite integral and know the basic properties of definite integrals;
25. know the Fundamental Theorem of Calculus and be able to use it for evaluating definite integrals and derivatives of integrals with variable limits of integration;
26. understand the concept of area of regions with curvilinear boundaries, be able to find area between curves;
27. be able to convert cartesian coordinates in polar coordinates and vice versa.
28. be able to sketch simple polar curves, e.g. $r = 1 - \cos\theta$ or $r = \sin(\theta/2)$.

Warnings

1. The above is intended as a MINIMAL list to be mastered in order to be reasonably sure of PASSING the examination.
2. Just because knowledge of a particular definition, formula or statement of a theorem is in the list of 'Learning outcomes' above does not guarantee that it will be on the examination paper. However, a good proportion will be, so they are worth knowing well.

Examination

The examination lasts for 2 hours. The rubric will state:
You should attempt all questions. Marks awarded are shown next to the questions.
Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.
Overall credit on this course will be computed using the algorithm:
20% for two tests and coursework, plus 80% for final exam.

Thomas Prellberg, September 2007