MAS115 Calculus I 2007-2008

Problem sheet for exercise class 9

- **Make sure you attend the excercise class that you have been assigned to!**
- Try to work on the problems first on your own. If you are stuck, ask for hints.
- The instructor and helper will be available for questions.
- Solutions will be available online by Friday.

Problem 1: **Making a simplifying substitution.** Evaluate

$$\int_0^{\sqrt{\ln 2}} 2xe^{x^2}\,dx\ .$$

Problem 2: **Completing the square.** Evaluate

$$\int \frac{d\theta}{\sqrt{2\theta-\theta^2}}\ .$$

Problem 3: **Using a trigonometric identity.** Evaluate

$$\int (\sin 3x\cos 2x-\cos 3x\sin 2x)dx\ .$$

Problem 4: **Eliminating a square root.** Evaluate

$$\int_{-\pi}^{0}\sqrt{1-\cos^2\theta}\,d\theta\ .$$

Problem 5: **Reducing an improper fraction.** Evaluate

$$\int_{\sqrt{2}}^{3}\frac{2x^3}{x^2-1}dx\ .$$

Problem 6: **Separating a fraction.** Evaluate

$$\int \frac{1-x}{\sqrt{1-x^2}}dx\ .$$

Problem 7: **Multiplying by 1.** Evaluate

$$\int \frac{1}{1+\sin x}dx\ .$$

Prize Question: **The best correct solution submitted to me on or before December 10 will be rewarded with a cash prize.** Evaluate

$$\int_0^1 \frac{x}{\sqrt{2\pi\alpha^3(1-\alpha)}}\exp\left(-\frac{x^2}{2\alpha(1-\alpha)}\right)d\alpha\ .$$

Thomas Prellberg, November 2007

Problem 1:

$$\int_0^{\sqrt{\ln 2}} 2xe^{x^2}\,dx;\ \begin{bmatrix} u = x^2 \\ du = 2x\,dx \\ x = 0 \Rightarrow u = 0,\ x = \sqrt{\ln 2} \Rightarrow u = \ln 2 \end{bmatrix} \rightarrow \int_0^{\ln 2} e^u\,du = [e^u]_0^{\ln 2} = e^{\ln 2} - e^0 = 2 - 1 = 1$$

Problem 2:

$$\int \frac{d\theta}{\sqrt{2\theta - \theta^2}} = \int \frac{d\theta}{\sqrt{1 - (\theta - 1)^2}};\ \begin{bmatrix} u = \theta - 1 \\ du = d\theta \end{bmatrix} \rightarrow \int \frac{du}{\sqrt{1 - u^2}} = \sin^{-1} u + C = \sin^{-1}(\theta - 1) + C$$

Problem 3:

$$\int (\sin 3x \cos 2x - \cos 3x \sin 2x)\,dx = \int \sin(3x - 2x)\,dx = \int \sin x\,dx = -\cos x + C$$

Problem 4:

$$\int_{-\pi}^{0} \sqrt{1 - \cos^2\theta}\,d\theta = \int_{-\pi}^{0} |\sin\theta|\,d\theta;\ \begin{bmatrix} \sin\theta \le 0 \\ \text{for } -\pi \le \theta \le 0 \end{bmatrix} \rightarrow \int_{-\pi}^{0} -\sin\theta\,d\theta = [\cos\theta]_{-\pi}^{0} = \cos 0 - \cos(-\pi)$$
$$= 1 - (-1) = 2$$

Problem 5:

$$\int_{\sqrt{2}}^{3} \frac{2x^3}{x^2 - 1}\,dx = \int_{\sqrt{2}}^{3} \left(2x + \frac{2x}{x^2 - 1}\right)dx = [x^2 + \ln|x^2 - 1|]_{\sqrt{2}}^{3} = (9 + \ln 8) - (2 + \ln 1) = 7 + \ln 8$$

Problem 6:

$$\int \frac{1 - x}{\sqrt{1 - x^2}}\,dx = \int \frac{dx}{\sqrt{1 - x^2}} - \int \frac{x\,dx}{\sqrt{1 - x^2}} = \sin^{-1} x + \sqrt{1 - x^2} + C$$

Problem 7:

$$\int \frac{dx}{1 + \sin x} = \int \frac{(1 - \sin x)}{(1 - \sin^2 x)}\,dx = \int \frac{(1 - \sin x)}{\cos^2 x}\,dx = \int (\sec^2 x - \sec x \tan x)\,dx = \tan x - \sec x + C$$