MAS115 Calculus I 2007-2008

Problem sheet for exercise class 7

- **Make sure you attend the excercise class that you have been assigned to!**

- The instructor will present the starred problems in class.

- You should then work on the other problems on your own.

- The instructor and helper will be available for questions.

- Solutions will be available online by Friday.

Problem 1:(*) a. Evaluate
$$\lim_{n\to\infty} \frac{1^5 + 2^5 + 3^5 + \ldots + n^5}{n^6}$$
by showing that the limit is
$$\int_0^1 x^5 dx$$
and evaluating the integral.

b. Evaluate
$$\lim_{n\to\infty} \frac{1^3 + 2^3 + 3^3 + \ldots + n^3}{n^4} \ .$$

Problem 2: Which formula is not equivalent to the other two?

a. $\sum_{j=2}^{4} \frac{(-1)^{j-1}}{j-1}$

b. $\sum_{k=0}^{2} \frac{(-1)^k}{k+1}$

c. $\sum_{l=-1}^{1} \frac{(-1)^l}{l+2}$

Problem 3: L'Hopital's rule does not help with the following limits. Find them some other way:

a. $\lim_{x\to\infty} \frac{\sqrt{x+5}}{\sqrt{x}+5}$

b. $\lim_{x\to\infty} \frac{2x}{x+7\sqrt{x}}$

Extra: Let $f(x)$, $g(x)$ be two continuously differentiable functions satisfying the relationships $f'(x) = g(x)$ and $f''(x) = -f(x)$. Let $h(x) = f^2(x) + g^2(x)$. If $h(0) = 5$, fing $h(10)$.

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