MAS115 Calculus I 2007-2008

Problem sheet for exercise class 6

- **Make sure you attend the excercise class that you have been assigned to!**
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online by Friday.

Strategy for Graphing $y = f(x)$

1. Identify the domain of f and any symmetries the curve may have.
2. Find y' and y''.
3. Find the critical points of f, and identify the function's behavior at each one.
4. Find where the curve is increasing and where it is decreasing.
5. Find the points of inflection, if any occur, and determine the concavity of the curve.
6. Identify any asymptotes.
7. Plot key points, such as the intercepts and the points found in Steps 3–5, and sketch the curve.

(*) Problem 1: Sketch the graph of $f(x) = \frac{(x+1)^2}{1+x^2}$.

Problem 2: Sketch the graph of $f(x) = \frac{x^3}{3x^2+1}$.

Problem 3: The sum of two non-negative numbers is 20. Find the numbers

      a. if the product of one number and the square root of the other is to be as large as possible.

      b. if one number plus the square root of the other is to be as large as possible.

Extra: The family of straight lines $y = ax + b$ (a, b arbitrary constants) can be characterised by the relation $y'' = 0$. Find a similar relation satisfied by the family of all circles

$$(x - h)^2 + (y - h)^2 = r^2 \, ,$$

where h and r are arbitrary constants.

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