MAS115 Calculus I 2007-2008

Problem sheet for exercise class 5

- **Make sure you attend the excercise class that you have been assigned to!**
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online by Friday.

Problem 1: [2007 exam questions]

a. State the definition of the derivative of the function $f(x)$ with respect to the variable x.

b. Given

$$\lim_{x\to 0}\frac{\cos x - 1}{x} = 0 \quad \text{and} \quad \lim_{x\to 0}\frac{\sin x}{x} = 1\,,$$

differentiate from first principles $f(x) = \cos x$.

(*) Problem 2: Does any tangent to the curve $y = \sqrt{x}$ cross the x-axis at $x = -1$? If so, find an equation for the line and the point of tangency. If not, why not?

Problem 3: Is there anything special about the tangents to the curves $y^2 = x^3$ and $2x^2 + 3y^2 = 5$ at the points $(1, \pm 1)$? Give reasons for the answer.

Extra: Suppose that a function f satisfies the following conditions for all real values of x and y:

i. $f(x + y) = f(x)f(y)$.

ii. $f(x) = 1 + xg(x)$, where $\lim_{x\to 0} g(x) = 1$.

Show that the derivative $f'(x)$ exists at every value of x and that $f'(x) = f(x)$.

Thomas Prellberg, October 2007