MAS115 Calculus I 2007-2008

Problem sheet for exercise class 4

- **Make sure you attend the excercise class that you have been assigned to!**
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online by Friday.

Problem 1: **Continuity.**

(*) a. Can $f(x) = x(x^2-1)/|x^2-1|$ be extended to be continuous at $x = 1$ or $x = -1$? Give reasons for your answers.

b. For what value of a is   [2007 exam questions]

$$f(x) = \begin{cases} x^2 - 1 \,, & x < 3 \\ 2ax \,, & x \geq 3 \end{cases}$$

continuous at every x?

Problem 2: **Limits and continuity.** Which of the following statements are true and which false? If true, say why; if false, give a counterexample (that is, an example confirming the falsehood).

a. If f is continuous at a, then so is $|f|$.

b. If $|f|$ is continuous at a, then so is f.

Problem 3: **The Intermediate Value Theorem.**   [2007 exam questions]

a. What are the hypotheses and conclusions of the Intermediate Value Theorem?

b. Using the Intermediate Value Theorem, explain why the equation

$$\cos x = x$$

has at least one solution.

Extra: **A function continuous at only one point.** Let

$$f(x) = \begin{cases} x \,, & \text{if } x \text{ is rational} \\ 0 \,, & \text{if } x \text{ is irrational.} \end{cases}$$

a. Show that f is continuous at $x = 0$.

b. Use the fact that every nonempty open interval of real numbers contains both rational and irrational numbers to show that f is not continuous at any nonzero value of x.

Thomas Prellberg, October 2007