MAS115 Calculus I 2007-2008

Problem sheet for exercise class 3

- **Make sure you attend the excercise class that you have been assigned to!**
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online by Friday.

(*) Problem 1: **Two wrong statements about limits.** Show by example that the following statements are wrong.

(a) The number L is the limit of $f(x)$ as x approaches x_0 if $f(x)$ gets closer to L as x approaches x_0.

(b) The number L is the limit of $f(x)$ as x approaches x_0 if, given any $\epsilon > 0$, there exists a value of x for which $|f(x) - L| < \epsilon$.

Explain why the functions in your examples do not have the given value of L as a limit as $x \to x_0$.

Problem 2: Compute the following limits: [2007 exam questions]

$$\text{(a)} \quad \lim_{x\to 9} \frac{x-9}{\sqrt{x+7}-4}\,, \qquad \text{(b)} \quad \lim_{u\to 3} \frac{u^3-27}{u^4-81}\,, \qquad \text{(c)} \quad \lim_{x\to 0} \frac{6x+6x\cos(6x)}{\sin(6x)\cos(6x)}\,.$$

Problem 3: Use the graph of the greatest integer function $y = \lfloor x \rfloor$ to determine the limits

$$\text{(a)} \quad \lim_{\theta\to 3^+} \frac{\lfloor\theta\rfloor}{\theta}\,, \quad \lim_{\theta\to 3^-} \frac{\lfloor\theta\rfloor}{\theta}\,, \qquad \text{(b)} \quad \lim_{t\to 4^+} (t - \lfloor t \rfloor)\,, \qquad \lim_{t\to 4^-} (t - \lfloor t \rfloor)\,.$$

Extra: **Roots of a quadratic equation that is almost linear.** The equation $ax^2+2x-1=0$, where a is a constant, has two roots if $a > -1$ and $a \neq 0$, one positive and one negative:

$$r_+(a) = \frac{-1+\sqrt{1+a}}{a}\,, \quad r_-(a) = \frac{-1-\sqrt{1+a}}{a}\,.$$

(a) What happens to $r_+(a)$ as $a \to 0$? As $a \to -1^+$?

(b) What happens to $r_-(a)$ as $a \to 0$? As $a \to -1^+$?

(c) Support your conclusions by graphing $r_+(a)$ and $r_-(a)$ as functions of a. Describe what you see.

Thomas Prellberg, October 2007