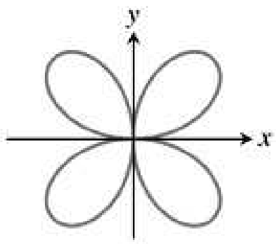
MAS115 Calculus I 2007-2008

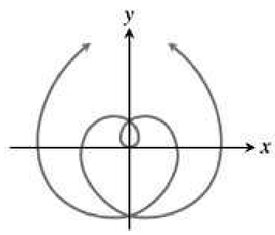
Problem sheet for exercise class 10

Four-leaved rose



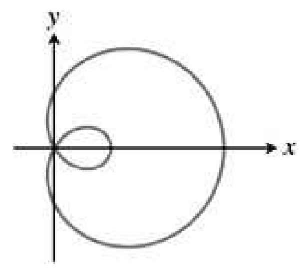
(*)

Spiral



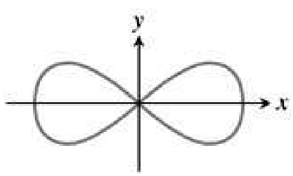
(*)

Limaçon

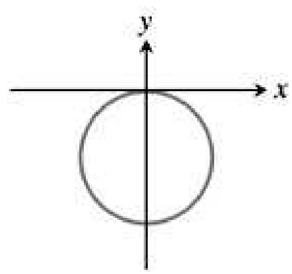


(*)

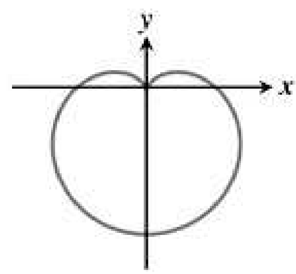
Lemniscate



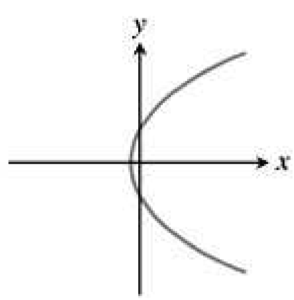
Circle



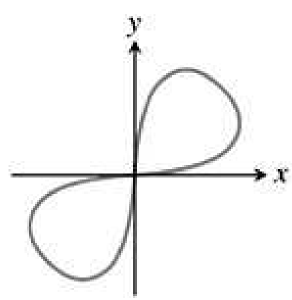
Cardioid



Parabola



Lemniscate



Problem 1: Match each of the eight graphs with one of the following equations.

a.	$r = \cos 2\theta$,	**b.**	$r\cos\theta = 1$,	**c.**	$r = \dfrac{6}{1 - 2\cos\theta}$,
d.	$r = \sin 2\theta$,	**e.**	$r = \theta$,	**f.**	$r^2 = \cos 2\theta$,
g.	$r = 1 + \cos\theta$,	**h.**	$r = 1 - \sin\theta$,	**i.**	$r = \dfrac{2}{1 - \cos\theta}$,
j.	$r^2 = \sin 2\theta$,	**k.**	$r = -\sin\theta$,	**l.**	$r = 2\cos\theta + 1$.

Problem 2: Show that the equations $x = r\cos\theta$, $y = r\sin\theta$ transform the polar equation

$$r = \frac{k}{1 + e\cos\theta}$$

into the Cartesian equation

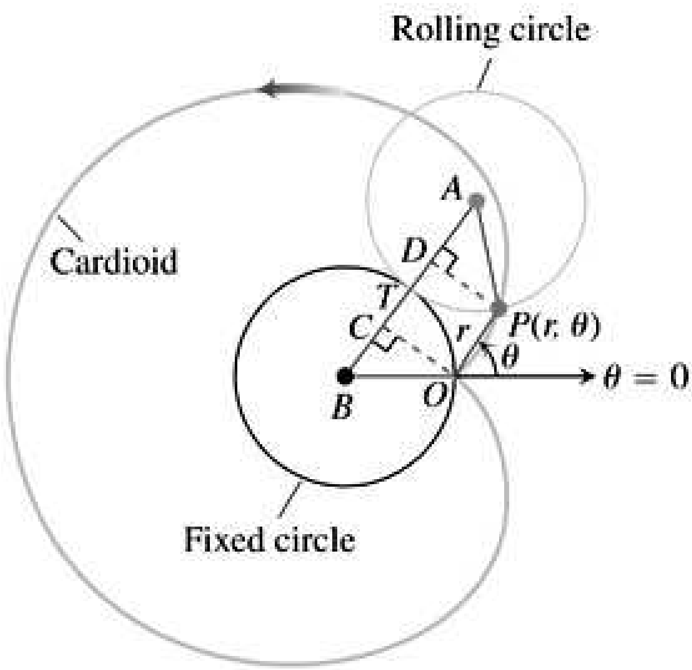
$$(1 - e^2)x^2 + y^2 + 2kex - k^2 = 0 \; .$$

Problem 3: Find polar equations for the following four circles. Sketch each circle in the coordinate plane and label it with both its Cartesian and polar equations.

a. $x^2 + y^2 + 5y = 0$, **b.** $x^2 + y^2 - 2y = 0$,

c. $x^2 + y^2 - 3x = 0$, **d.** $x^2 + y^2 + 4x = 0$.

Extra: Show that if you roll a circle of radius a about another circle of radius a in the polar coordinate plane, the original point of contact P will trace a cardioid. (Hint: start by showing that $\angle OBC$ and $\angle PAD$ are equal to each other.)



Thomas Prellberg, December 2007