Queen Mary
University of London

B.Sc. EXAMINATION BY COURSE UNITS

MAS115 Calculus I — Sample Exam Paper

This is a sample paper showing the format of the exam.

The duration of this examination is 2 hours.

You should attempt all questions. Marks awarded are shown next to the questions. Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Candidates must not remove the question paper from the examination room.

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR

1. Marks are only awarded for the final answer, so indicate this answer clearly.

(a) [5 marks] Find the limit

$$\lim_{x\to 9} \frac{x-9}{\sqrt{x+7}-4} \, .$$

(b) [5 marks] Find the slope of the curve

$$4y^4 + 5x^8 = 3y + 6x$$

at the point $(1, 1)$.

(c) [5 marks] Determine the point(s) at which the function

$$f(x) = 9\csc(6x)$$

is continuous.

(d) [5 marks] Use the relation $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ to find the limit

$$\lim_{x\to 0} \frac{6x + 6x\cos(6x)}{\sin(6x)\cos(6x)} \, .$$

(e) [5 marks] Find the first derivative of the function

$$y = \cos\left((8t+9)^{-4/7}\right) \, .$$

(f) [5 marks] Evaluate

$$\int_{1/5}^{3} 12x\ln(5x)dx \, .$$

(g) [5 marks] Evaluate

$$\int_{-\pi}^{0} \sin(14x)\cos(5x)dx \, .$$

(h) [5 marks] Evaluate

$$\int 2\cdot 7^{2x+7}dx \, .$$

(i) [5 marks] Find the area enclosed by the two curves

$$9x^2 + y = 9$$

and

$$x^6 - y = 1 \, .$$

(j) [5 marks] Evaluate

$$\int_{14}^{\infty} \frac{6}{v^2 - v}dv \, .$$

[Next question overleaf]

2. [10 marks]

(a) State the definition of the derivative of the function $f(x)$ with respect to the variable x.

(b) Show that differentiability implies continuity:
If f has a derivative at $x = c$, then f is continuous at $x = c$.

3. [20 marks] Consider the curve $y = f(x)$ for the function

$$f(x) = \frac{(2+x)^2}{1+x} \ .$$

(a) Identify the domain of f and any symmetries the curve may have.

(b) Find $f'(x)$ and $f''(x)$.

(c) Find the critical points of f, and identify the function's behaviour at each one.

(d) Find where the curve is increasing and where it is decreasing.

(e) Find the points of inflection, if any occur, and determine the concavity of the curve.

(f) Identify any asymptotes.

(g) Plot key points, such as intercepts, critical points, and points of inflection, and sketch the curve.

4. [10 marks]

(a) What are the hypotheses and conclusions of the Intermediate Value Theorem?

(b) Using the Intermediate Value Theorem, explain why the equation

$$\cos x = x$$

has at least one solution.

5. [10 marks] Consider the curve

$$r^2 = 4\cos\theta \ .$$

(a) Identify any symmetries the curve may have.

(b) Make a short table of values and graph the curve.

(c) The area of the region between the origin and the curve $r = f(\theta)$, $\alpha \leq \theta \leq \beta$ is given by

$$A = \int_\alpha^\beta \frac{1}{2} r^2 d\theta \ .$$

Use this formula to compute the area of the region enclosed by the curve.

[End of examination paper]