Lie Superalgebras, Clifford Algebras, Induced Modules and Nilpotent Orbits

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Let $\mathfrak{g}$ be a classical simple Lie superalgebra. To every nilpotent orbit $\mathcal{O}$ in $\mathfrak{g}_0$ we associate a Clifford algebra over the field of rational functions on $\mathcal{O}$. We find the rank, $k(\mathcal{O})$ of the bilinear form defining this Clifford algebra, and deduce a lower bound on the multiplicity of a $U(\mathfrak{g})$-module with $\mathcal{O}$ or an orbital subvariety of $\mathcal{O}$ as associated variety. In some cases we obtain modules where the lower bound on multiplicity is attained using parabolic induction. The invariant $k(\mathcal{O})$ is in many cases, equal to the odd dimension of the orbit $G \cdot \mathcal{O}$ where G is a Lie supergroup with Lie superalgebra $\mathfrak{g}$.

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