Christian Elsholtz: Additive decompositions of the set of primes and new methods in prime number theory

This is a survey on Ostmann's problem. Ostmann asked whether there exist two sets A and B (with at least two elements each) so that their sumset A + Bequals the set of primes, for sufficiently large primes. Using a new version of the large sieve method I can show, that such sets A and B would need to have counting functions of size $N^{1/2+o(1)}$, whereas previously only a lower bound of $N^{o(1)}$ and an upper bound of $N^{1+o(1)}$ was known. This implies, for example, that the set of primes cannot be decomposed into three such sets. We also look at very thin sets of primes such as primes of the form $x^2 + y^4$ and show that underlying additive structures exist which are larger than one might have expected.

This talk will give a nontechnical survey of the underlying ideas and show how a new type of the large sieve method and combinatorial counting arguments (including graph theory) can be applied to such problems.