Fischer spaces and Lie algebras

Hans Cuypers (Eindhoven)

A Fischer space is a partial linear space (P,L) in which each line contains 3 points and two intersecting lines generate a subspace isomorphic to an affine or dual affine plane.

Starting from a Fischer space (P,L) one can construct an algebra on the vector space 2^P defined by the rule that

$$p * q = \begin{cases} p + q + r & \text{if } \{p, q, r\} \text{ is a line,} \\ 0 & \text{if } p \text{ and } q \text{ are not collinear or } p = q. \end{cases}$$

We study the structure of this algebra and determine the simple Lie algebras that arise from this construction.