Fair games and fixed points

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Fifty years ago, John Isbell made a conjecture. The context was many-player games (in the sense of von Neumann and Morgenstern), and the problem can be translated into one about maximal intersecting families of sets, but it is really a question about permutation groups: it asserts that if the degree of a finite transitive permutation group G is the product of a small odd number and a very large power of 2, then G contains fixed-point-free elements of 2-power order. This problem is still unsolved, but has been generalised; recently Eleonora Crestani and Pablo Spiga have demonstrated that the strongest such conjecture is false.

I will talk about the background to the conjecture and survey some results on it and some related results; time permitting, I will say something about the proof by Crestani and Spiga, which uses a certain thin pro-*p*-group.