The local-global approach to Diophantine equations

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The study of whole-number solutions of polynomial equations has attracted mathematicians for thousands of years, yet we are still far from understanding anything but the simplest equations. One approach is to look at "local" information: that is, information over the real numbers and modulo prime powers; and to try to deduce information about the "global" solutions: that is, solutions in integers or rational numbers. For example, Hasse proved that a quadratic form in any number of variables has a rational zero if, and only if, it has zeros in the real numbers and modulo each prime power. More general equations do not satisfy this "Hasse principle"; I will describe ways to understand this failure.