Peter Cameron: *Factorising the cycle index of a permutation group*

Merino, de Mier and Noy showed that the bivariate Tutte polynomial of a matroid is irreducible if the matroid is connected. Simeon Ball suggested that a similar result might hold for the cycle index of a permutation group.

The *cycle index* of G is, apart from a constant factor,

$$S(G) = \sum_{g\in G} s_1^{c_1(g)} \cdots s_n^{c_n(g)},$$

where $s_1,\ldots,s_n$ are indeterminates and $c_i(g)$ is the number of cycles of length i in the cycle decomposition of g. It is known that $Z(G_1 \times G_2) = Z(G_1)Z(G_2)$ if the direct product acts on the disjoint union of the permutation domains for the factors. Also, if the cycle index for G is reducible, so is that for the wreath product $H \wr G$ in its imprimitive action. There are also two infinite families of transitive groups, and at least one sporadic intransitive group, for which the cycle index is reducible.

We conjecture that the cycle index of a primitive group is irreducible.

I will talk about some aspects of this problem. The upshot is that the situation is much more complicated than for the Tutte polynomial.