Intervals

Robert Woodrow (Calgary)

The family of intervals of a binary structure on a set S satisfies well known properties:

1. The empty set, the set S and every singleton is an interval.
2. The intersection of a family of intervals is an interval.
3. If X and Y are intervals with non empty intersection, the union is an interval.
4. If X and Y are intervals and $Y \setminus X$ is non empty then $X \setminus Y$ is an interval.
5. The union of an up-directed family of intervals is an interval.

A family of subsets of S with these properties is called *weakly partitive*.

An interval X is called *strong* provided that for each interval Y, if the intersection of X and Y is non-empty then Y is a subset of X or Y contains X. Using the notion of strong interval, and a study of the characteristics of elements of a weakly partitive family Pierre Ille and I gave a proof in [1] of his result that given a weakly partitive family I on a set S there is a binary structure on S whose intervals are exactly the elements of I.

References

[1] Weakly partitive families on infinite sets, Pierre Ille and Robert E. Woodrow, *Contributions to Discrete Mathematics*, Vol 4, Number 1, 2009, pp. 54–79.