A problem of Erdős and Sós on 3-graphs

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We show that for every $\varepsilon > 0$ there exist $\delta > 0$ and $n_0 \in \mathbb{N}$ such that every 3-uniform hypergraph on $n \geq n_0$ vertices with the property that every k-vertex subset, where $k \geq \delta n$, induces at least $\left(\frac{1}{4} + \varepsilon\right)\binom{k}{3}$ edges, contains K_4^- as a subgraph, where K_4^- is the 3-uniform hypergraph on 4 vertices with 3 edges. This question was originally raised by Erdős and Sós. The constant $1/4$ is the best possible. This is a joint work with Roman Glebov and Dan Kral.