The phase transition in random graphs – a simple proof

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The classical result of Erdős and Rényi shows that the random graph $G(n,p)$ experiences sharp phase transition around $p = 1/n$ – for any $\varepsilon > 0$ and $p = (1-\varepsilon)/n$, all connected components of $G(n,p)$ are typically of size $O(\log n)$, while for $p = (1+\varepsilon)/n$, with high probability there exists a connected component of size linear in n. We provide a very simple proof of this fundamental result; in fact, we prove that in the supercritical regime $p = (1+\varepsilon)/n$, the random graph $G(n,p)$ contains typically a path of linear length. We also discuss applications of our technique to other random graph models and to positional games.

Joint work with M. Krivelelvich