Synchronizing non-uniform maps

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A finite deterministic automaton is *synchronizing* if there is a sequence of transitions which ends up at the same state no matter which state we start from.

There has been a lot of interest in the case where the transitions of the automaton consist of the generators of a permutation group G and a single non-permutation f: I will say that *G synchronizes f* if $\langle G, f \rangle$ is synchronizing.

It is known that a permutation group which synchronizes every non-permutation is primitive, but the converse is false. An important theorem of Rystsov asserts that a group of degree n is primitive if and only if it synchronizes every map of rank $n-1$; also, a primitive group synchronizes any map of rank 2. João Araújo has conjectured that a primitive group synchronizes every *non-uniform* map (one whose kernel classes do not all have the same size).

I will report on some progress on this conjecture that João and I made last month, including an improvement of Rystsov's theorem from $n-1$ to $n-2$.