Q_2-free families in the Boolean lattice

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For a family of subsets of $\{1,\ldots,n\}$, ordered by inclusion and a partially-ordered set P, we say that the family is P-free if it does not contain a subposet isomorphic to P. We are interested in finding $\mathrm{ex}(n,P)$, the largest size of a P-free family of subsets of $[n]$. It is conjectured that, for any fixed P, this quantity is $(k+o(1))\binom{n}{2}$ for some fixed integer k, depending only on P.

Recently, Boris Bukh has verified the conjecture for P which are in a "tree shape". There are some other small posets P for which the conjecture has been verified. The smallest for which it is unknown is Q_2, the Boolean lattice on two elements. We will discuss the best-known upper bound for $\mathrm{ex}(n,Q_2)$ and an interesting open problem on graph theory that, if solved, would improve this bound. This is joint work with Maria Axenovich, Iowa State University and Jacob Manske, Texas State University.