Peter Cameron: *Algebraic properties of chromatic roots*

This is a report of a working group at the Isaac Newton Institute in the first half of 2008, during the program on *Combinatorics and Statistical Mechanics*. The chromatic polynomial of a graph G is the (unique) polynomial $p(x)$ such that, for any positive integer q, the number of proper vertex-colourings of G with q colours is $p(q)$. It is a monic integer polynomial, and so its roots are algebraic integers. David Wallace asked the question: *which algebraic integers occur as chromatic roots?*

Computational evidence suggests the wild speculation that the "typical" behaviour of the chromatic polynomial is that it is a product of factors of degree 1 and a single irreducible polynomial of degree greater than 1 whose Galois group is the symmetric group. (Of course, not every chromatic polynomial has this form!)

In a different direction, we came up with two conjectures:

- For any algebraic integer α, there is a natural number n such that $\alpha + n$ is a chromatic root.
- If α is a chromatic root, then so is $n\alpha$ for any natural number n.

I will discuss these speculations, explaining all the background, and describe the few positive results that we have been able to obtain.