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LOCATING CRITICAL LINES IN HIGH-VOLTAGE ELECTRICAL POWER GRIDS

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Electrical power grids are among the infrastructures that are attracting a great deal of attention because of their intrinsic criticality. Here we analyze the topological vulnerability and improvability of the spanish 400 kV, the french 400 kV and the italian 380 kV power transmission grids. For each network we detect the most critical lines and suggest how to improve the connectivity.

Keywords: Complex networks; electric power grids; vulnerability; improvements.

1. Introduction

Nowadays the issue of vulnerability and protection of critical infrastructure is attracting a great deal of attention among scientific researchers. One of the most used approaches is that of complex networks. A critical infrastructure (e.g. a power grid) is represented as a graph in which nodes represent the main components of the network (e.g. power plants or trasmission substations) and edges are the physical connections among them (i.e. the electric lines in the above example). Following this approach, several scientific works have been produced with the purpose of analyzing error and attack resilience of both artificially generated topologies and real world networks.

In literature, failures and attacks have been simulated as the removal of a certain percentage either of nodes [1–5] or of edges [2, 6] of the network.

In particular, a general method to find the critical components of a network has been introduced in Ref. [7]. There, the authors assume that the performance of a network G can be measured by a single variable $\Phi(G) > 0$, and propose to use the variation in the performance to spot the critical components of G.

The performance of G is evaluated from the network structure as: $\Phi(G) \equiv E(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}}$, where $E(G)$ is the global efficiency of G defined from the shortest path lengths $\{d_{ij}\}$ (number of steps) from node i to node j [8].

Different from the characteristic path length [1], the efficiency E is a well-defined quantity also for non-connected graphs [3, 8]. Indicating with D the set of possible damages, and with $DAMAGE(G, d)$ a map that gives the network resulting from G after a damage $d \in D$ (for instance after the removal of single nodes, or single edges, or couples of edges), the importance of the damage d is measured by the relative drop in performance $\Delta\Phi^-/\Phi$, with $\Delta\Phi^- = \Phi[G] - \Phi[DAMAGE(G, d)]$.

The critical damages are those that maximize $\Delta\Phi^-/\Phi$, and the larger $\Delta\Phi^-/\Phi$ the higher is the vulnerability of G under the class of damages D. Locating the critical components in an infrastructure is a very important task. However, also finding the most useful possible improvements is fundamental in real world networks. In an analogous way, given a set of improvements I (e.g. the addition of edges), and defined for any improvement $i \in I$ the map $IMPROVE(G, i)$ that gives the network obtained after the improvement i, the importance of the improvement i can be defined as $\Delta\Phi^+/\Phi$, with $\Delta\Phi^+ = \Phi[IMPROVE(G, d)] - \Phi[G]$. The best improvements are those that maximize $\Delta\Phi^+/\Phi$, and the larger $\Delta\Phi^+/\Phi$ the higher is the improvability of G under the class of improvements I.

In this paper we apply the method of Ref. [7] to study the topological vulnerability and improvability of three different power grids: (1) the spanish 400 kV electric network, with $N_S = 98$ substations and $K_S = 175$ lines; (2) the french 400 kV electric network, with $N_F = 146$ and $K_F = 223$; (3) the italian 380 kV electric network, with $N_I = 127$ and $K_I = 171$. The power grids are the same studied by Rosato *et al.* in Ref. [9].

We consider three different classes of damages D, namely the removal of single edges, couples of edges and triples of edges, and one class of improvements, namely the addition of single edges. An alternative possibility, not explored in this paper, is the removal of nodes.

The results of the analysis are shown in Sec. 2. In Sec. 3 we study the damage probability distribution obtained by removing a higher number of edges (from 5 to 25) and, in Sec. 4, we draw some conclusions.

2. Critical Damages and Improvements

The three power grids studied are shown in Figs. 1–3 [10].

In Table 1 we report the four most critical edges, obtained by considering single-edge removals. The italian network shows to be the most vulnerable one: the removal of a single edge is sufficient to generate a drop in the efficiency close to 5%, while in the french and in the spanish power grids the drop in the efficiency is limited to about 3%. The reason for this difference has to be found in the stretched shape of the italian peninsula (and consequently of the related power grid), in which, especially in the extreme South, nodes are poorly linked (See Fig. 3). For instance

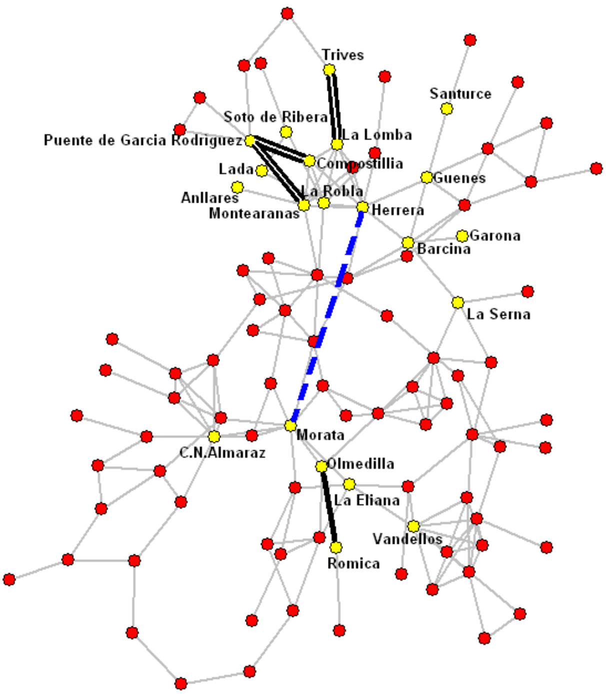

Fig. 1. The spanish 400kV power grid with the most vulnerable edge (black thick line), the most vulnerable triple of edges (black double lines), and the edge that, if added to the network, improves the performance of the system best (dashed blue line). Nodes reported in tables are marked with a lighter color and with a label indicating the name of their respective substations.

the removal of a single edge, as the line connecting Laino and Rossano, is sufficient to isolate seven nodes from the rest of the network.

Analogous observations are valid when the class of possible damages is restricted to couples of edges (Table 2). By removing the line connecting Laino and Rossano, and that connecting Vignole B. and La Spezia, the efficiency of the italian power grid experiences a drop of 8.32%. It is worthy noting that not always the joint removal of the two most important edge of Table 1 is among the most damaging strategies for removals of couples of edges. In fact, if we remove both the Laino–Rossano and the Rossano–Scandale lines from the italian grid, we found a drop in efficiency only equal to 5.2%, and such a removal ranks only 220th in the list.

The differences between the italian and the other two networks is even more evident when three edges are removed jointly (Table 3). In fact, with removals of triples of nodes, the italian power grid can be broken into two different subgraphs of approximately the same size, thus decreasing the efficiency by 31%. This result is in agreement with those obtained in Ref. [9] by means of a spectral bisection method.

Bisection methods allow to find the division of the vertices of a graph into two nearly equally-sized vertex subsets G_1 and G_2, with the minimum number of edges running between them. The edges connecting the two subgraphs found by the

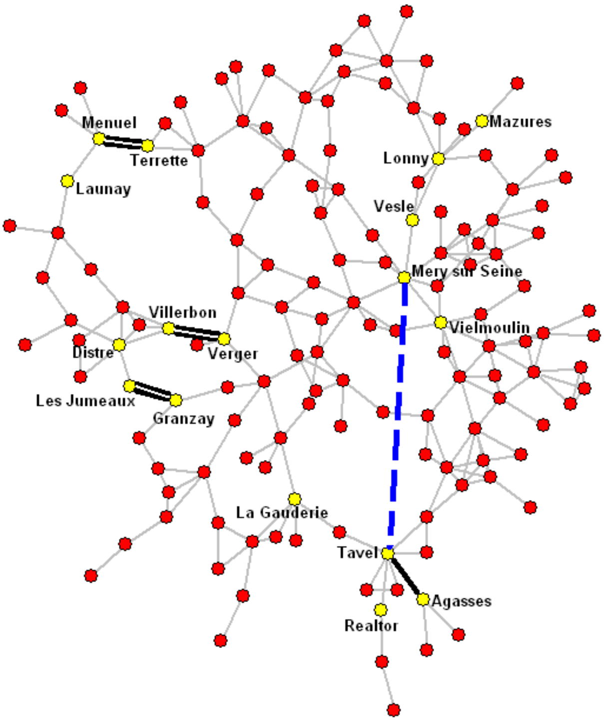

Fig. 2. The french 400kV power grid with the most vulnerable edge (black thick line), the most vulnerable triple of edges (black double lines), and the edge that, if added to the network, improves the performance of the system best (dashed blue line). Nodes reported in tables are marked with a lighter color and with a label indicating the name of their respective substations.

authors of Ref. [9] for the italian grid, are three and are those at the 3rd rank in Table 3.

Finally, the removal of the three most important edges in Table 1 is not equivalent to perform an important three-edges removal: the joint removal of the lines Mery sur Seine–Vesle, Tavel–Agasses, and Tavel–Realtor is at the 20th place in the rank for the french grid and the joint removal of the lines Laino–Rossano, Rizziconi–Scandale, and Rossano–Scandale ranks 33302nd for the italian grid. The most important couple of edges of Table 2 is not either meaningful for the three-edges removals: the most important three-edges removal that involves the couple of edges Laino–Rossano and Vignole B.-La Spezia ranks only 65th.

Turning now to the improvements, we report in Table 4 the best addition of single edges. The italian power grid not only is the most vulnerable network, but is also the most improvable one. For instance, the addition of a single edge between Piacenza and Benevento causes an improvement in efficiency of almost 8%. Obviously, it is not easy to realize a physical connection between two distant substations: a careful trade-off between realization cost and efficiency improvement should be taken into account. However, in this paper we ignore it, dealing only with topological connections.

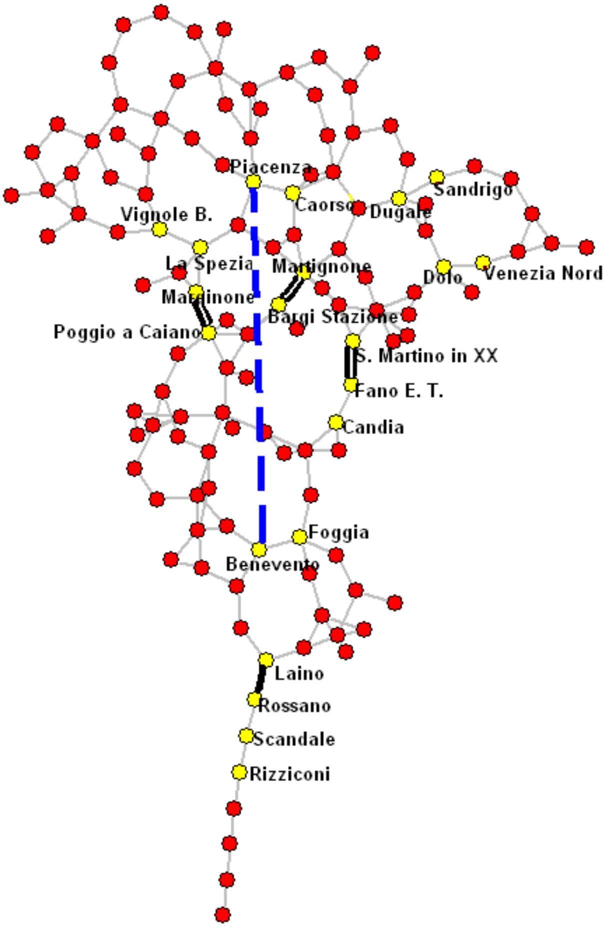

Fig. 3. The italian 380kV power grid with the most vulnerable edge (black thick line), the most vulnerable triple of edges (black double lines), and the edge that, if added to the network, improves the performance of the system best (dashed blue line). See tables. Nodes reported in tables are marked with a lighter color and with a label indicating the name of their respective substations.

3. A Glimpse to Distributions

An evaluation of all the possible multiple removals is too much time-consuming for a number of edges larger than 2 or 3. In this section we consider five different classes of damage, namely the removal of $m = 5, 10, 15, 20, 25$ edges. For each class of damage we calculate the drop in the efficiency for 10000 randomly selected different cases, out of the total number of possibilities which is of the order of K^m (with K being the number of lines in the network).

In Figs. 4 and 5 we report the damage distributions, i.e. the probability of causing a certain damage. The distribution is gaussian both for the spanish and the french networks. Obviously, when the number of removed edges increases, the probability of causing high damages grows, i.e. the distribution curves move to

Table 1. The four most critical lines and the relative damage (in percentage).

	S-400kV	$\frac{\Delta\Phi^-}{\Phi}$	F-400kV	$\frac{\Delta\Phi^-}{\Phi}$	I-380kV	$\frac{\Delta\Phi^-}{\Phi}$
1.	Olmed.-Romica	3.22	Tavel-Agasses	2.96	Laino-Ross.	5.01
2.	Gueñes-Santurce	3.03	Tavel-Realtor	2.84	Rossano-Scand.	4.18
3.	Barcina-La Serna	2.00	Mery sur S.-Vesle	2.50	Rizz.-Scand.	3.42
4.	Barcina-Garoña	1.90	Mazures-Lonny	2.22	Vign.B.-La Sp.	3.39

Table 2. The four most critical couples of lines and the relative damage (in percentage).

	S-400kV	$\frac{\Delta\Phi^-}{\Phi}$	F-400kV	$\frac{\Delta\Phi^-}{\Phi}$	I-380kV	$\frac{\Delta\Phi^-}{\Phi}$
1.	Olmed.-Romica Gueñes-Santurce	6.22	Tavel-Agasses Tavel-Realtor	5.67	Laino-Ross. Vign. B.-La Sp.	8.32
2.	Olmed.-Romica Barcina-La Serna	5.22	Mery sur S.-Vesle Tavel-Agasses	5.43	Dugale-Sandrigo Dolo-Ven. N.	8.20
3.	Barcina-Garoña Olmed.-Romica	5.10	Mery sur S.-Vesle Tavel-Realtor	5.31	Mart.-Bargi St. Magin.-Pogg. a C.	7.72
4.	Mont.-Anllares Olmed.-Romica	5.04	Mazures-Lonny Tavel-Agasses	5.16	Laino-Ross. Magin.-Pogg. a C.	7.71

Table 3. Critical triples of lines and relative damage (in percentage).

	S-400kV	$\frac{\Delta\Phi^-}{\Phi}$	F-400kV	$\frac{\Delta\Phi^-}{\Phi}$	I-380kV	$\frac{\Delta\Phi^-}{\Phi}$
1.	P.de G.R.-Compost. P.de G.R.-Mont. Trives-La Lomba	8.72	Menuel-Terrette Vill.-Verger Granzay-Les Jum.	14.23	Mart.-Bargi St. S.M. in XX-Fano Marg.-Pogg. a C.	30.96
2.	Olmed.-Romica Soto de R.-La Robla Lada-La Robla	8.09	Menuel-Terrette Distre-Les Jum. Marg.-Pogg. a C.	13.57	Mart.-Bargi St. Vill.-Verger Fano-Candia	30.73
3.	Olmed.-Romica Barcina-La Serna Gueñes-Sant.	8.06	Menuel-Launay Vill.-Verger Granzay-Les Jum.	12.43	S.M. in XX-Fano Marg.-Pogg. a C. Bargi St.-Calenz.	30.21
4.	Olmed.-Romica Barcina-Garoña Gueñes-Santurce	8.05	Launay-Domloup Vill.-Verger Granzay-Les Jum.	11.84	Marg.-Pogg. a C. Bargi St.-Calenz. Fano-Candia	29.93

Table 4. The four best improvement and the relative increase in performance (in percentage).

	S-400kV	$\frac{\Delta\Phi^+}{\Phi}$	F-400kV	$\frac{\Delta\Phi^+}{\Phi}$	I-380kV	$\frac{\Delta\Phi^+}{\Phi}$
1.	Herr.-Morata	4.39	Mery s. S. -Tavel	4.41	Piac.-Benev.	7.93
2.	Herr.-Vandellos	4.15	Distre-Mery s. S.	4.32	Piac.-Foggia	7.89
3.	Herr.-La Eliana	4.14	Distre-Vielmoulin	4.24	Caorso-Benev.	7.87
4.	Herr.-C. N. Alm.	4.05	Mery s. S. -La Gaud.	4.21	Caorso-Foggia	7.77

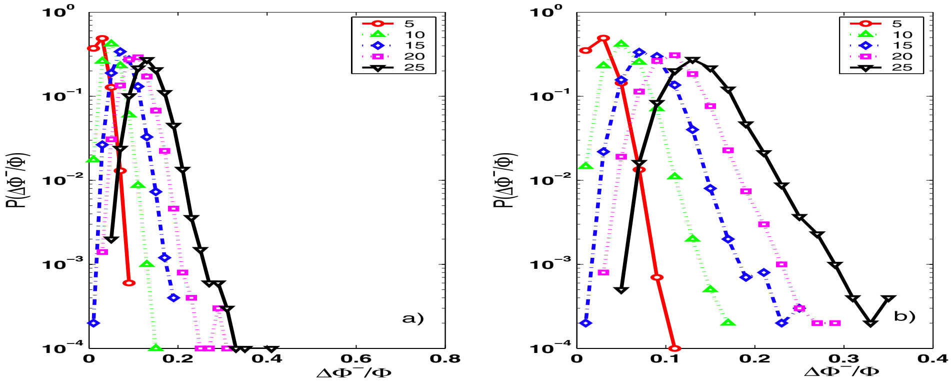

Fig. 4. Damage distribution in log-linear scale for a) the spanish and b) the french 400 kV power grid, and for a number of removed edges equal to 5,10,15,20,25.

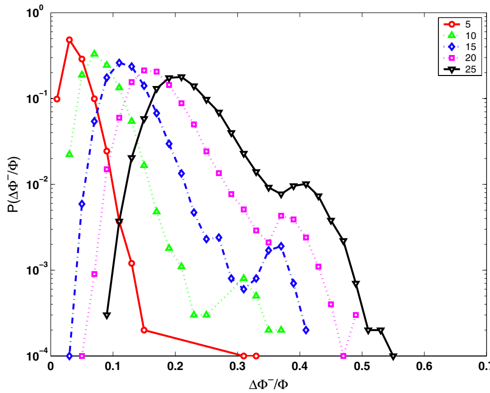

Fig. 5. Damage distribution in log-linear scale for the italian 380kV power grid, and for a number of removed edges equal to 5,10,15,20,25.

the right side of figures. An interesting difference can be noticed for the italian power grid, where the damage distribution is bimodal, especially for a number of removed edges in the range from 10 to 20, and can be fitted by the sum of two gaussians. Again, the reason for this particular behavior has to be found in the particular shape of the italian power grid. In fact when a certain number of edges are removed, the damage with the highest probability is low, but there is also a high probability to break the entire network at least into two big subgraphs (as in the case of the removal of the most important triples of edges in Table 3). It is not accidentally that the peaks of the secondary gaussian curves are close to the value that the drop in efficiency has when the removal concerns only the minimum number of nodes (three) for breaking the network into two big subgraphs. Obviously, when a huge number of edges is removed, also other meaningful lines can happen to be removed and this explains why the peak moves to the right for increasing number of removals.

4. Conclusions

In this paper we have studied and compared the topological properties of three electric power grids. We have used a general method that allows to find the critical components of a graph, i.e. the nodes and edges that when removed affect the most the structure of the graph. The method also allows to find the edges that, if added to the network, generate the best improvements. Moreover, we have analyzed the damage distributions and related them to the shape and the connectivity of the networks. Results have shown that the stretched shape of the italian power grid makes it structurally very different from the other two systems. In fact, on one hand, it shows to be so vulnerable that the joint removal of only three edges is sufficient to break dramatically the network and to cause a drop in the efficiency close to 30%. On the other hand, it is also the network that can be improved most with the addition of a single edge.

Obviously, in order to design an improvement to real electrical networks, further aspects concerning both the modeling of dynamical cascading failures (see [5,11,12] and references therein), and the inclusion of euclidean distances between graph nodes, should be considered.

Acknowledgments

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