Title: Anomalous diffusion in a weakly chaotic dynamical system

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Background: Anomalous transport became a very important topic over the past couple of years [1]. This new, highly interdisciplinary field of research deals with deviations from simple Brownian motion in terms of non-Gaussian processes and non-Markovian dynamics. These dynamics lead to phenomena like sub- or superdiffusion, i.e., where particles spread out slower or faster, respectively, than compared to ordinary Brownian motion. Such phenomena can be modeled by concepts of advanced stochastic theory like continuous time random walks and fractional diffusion equations [2]. The theory has wide applications in many different fields of research such as, e.g., diffusion in biological systems, molecular diffusion in nanopores, and even in socio-economics [1].
A particularly challenging problem is to cross-link the widely applied stochastic theory of anomalous transport to the theory of chaotic dynamical systems [3]. This topic can best be explored by studying simple weakly chaotic maps defined on the whole real line. A paradigmatic example is the intermittent Pomeau-Manneville map, which can be used to generate the whole range of anomalous diffusion. In this model a non-trivial transition from normal diffusion to subdiffusion was found under parameter variation. This transition was understood by analytical approximations obtained both from stochastic theory and from dynamical systems theory by reproducing results from computer simulations [4].

Project description: Parameter-dependent diffusion in the more difficult case of the superdiffusive Pomeau-Manneville map should be explored. Here a non-trivial transition from normal to superdiffusion was conjectured. This may first be checked by performing computer simulations. Numerical results should then be compared with analytical approximations obtained from stochastic theory. An advanced method to study the diffusive properties of this map should be developed by using a more rigorous dynamical systems approach based on Lagrange-Chebychev polynomials. There is the possibility to collaborate with O.Bandtlow on more rigorously mathematical aspects of dynamical systems theory, and with A.V.Chechkin on advanced stochastic theory. This highly interdisciplinary project is right at the interface between stochastic theory, dynamical systems theory, statistical physics, and computer simulations.

References: (all available on the supervisor's homepage)

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