Queen Mary
University of London

B. Sc. Examination by course unit 2010

MTH4100 Calculus 1

Duration: 2 hours

Date and time: 13 May 2010, 10:00h–12:00h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): R. Klages, M. Jerrum

 TURN OVER

Question 1 [total: 40 marks]

(a) Determine the set of real numbers $x \in \mathbb{R}$ that satisfy the inequality

$$|4x - 3| \leq 8 \quad .$$

[4 marks]

(b) Find the limit

$$\lim_{h \to 0} \frac{\sqrt{13h + 1} - 1}{h} \, .$$

[4 marks]

(c) Find the limit

$$\lim_{x \to 3} \frac{2x - 6}{5x^2 - 45} \, .$$

[4 marks]

(d) Find the horizontal, vertical and oblique asymptotes, if any, of

$$f(x) = \frac{3x^2}{x - 2} \, .$$

[5 marks]

(e) Find the derivative $g'(x)$ of

$$g(x) = \frac{9x - 7}{3x + 1} \quad , \quad x \neq -\frac{1}{3} \quad .$$

[5 marks]

(f) Find the derivative $h'(x)$ of

$$h(x) = x \tan\left(8\sqrt{x}\right) + 2 \quad , \quad x \geq 0 \, .$$

[6 marks]

(g) Evaluate

$$\int \frac{1}{\sqrt{7x}(\sqrt{7x} + 3)} \, dx$$

for $x > 0$. Simplify your answer. [6 marks]

(h) Evaluate

$$\int_{1/2}^{3} 6x \ln(2x) \, dx \, .$$

Simplify your answer. [6 marks]

Question 2 [total: 30 marks] Consider the curve $y = f(x)$ for the function

$$f(x) = x^2 + \frac{2}{x} .$$

(a) Identify the domain of f and any symmetries the curve may have. [3 marks]

(b) Find $f'(x)$ and $f''(x)$. [2 marks]

(c) State the definition of a critical point. Find the critical points for f, and identify the function's behaviour at each one. [7 marks]

(d) Find where the curve is increasing and where it is decreasing. [3 marks]

(e) State the definition of an inflection point. Find the inflection points for f, if any occur, and determine the concavity of the curve. [8 marks]

(f) Identify any asymptotes. [2 marks]

(g) Plot key points, such as intercepts, critical points, and points of inflection, and sketch the curve. [5 marks]

Question 3 [total: 10 marks]

(a) State the definition of the derivative of the function $f(x)$ with respect to the variable x. [4 marks]

(b) Differentiate from first principles, that is, by using the above definition of the derivative, $f(x) = \dfrac{1}{x^2}$. [6 marks]

Question 4 [20 marks] For the function $f(x) = x + x^2$ over the interval $[0, 1]$, find a formula for the *upper sum* obtained by dividing the interval $[0, 1]$ into n equal subintervals. Then take the limit of this sum as $n \to \infty$ to calculate the area under the curve over $[0, 1]$.

End of Paper