

**B. Sc. Examination by course unit 2009**

**MTH4100 Calculus 1**

**Duration: 2 hours**

**Date and time: 11 May 2009, 10:00h–12:00h**

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Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): R. Klages, M. Jerrum

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**Question 1** [total: 40 marks]

(a) Find the limit

$$\lim_{t \rightarrow 5} \frac{t^2 + 3t - 40}{t^2 - 25} .$$

[5 marks]

(b) Find the limit

$$\lim_{x \rightarrow -4^+} (x + 5) \frac{|x + 4|}{x + 4} .$$

[5 marks]

(c) Define what is meant by even and odd functions. Then determine whether the function

$$f(x) = 3x^5 - 4x^2$$

is even, odd, or neither.

[5 marks]

(d) Find the first and second derivatives of

$$y = \frac{4x^5 + 8}{x^3} , \quad x \neq 0 .$$

[5 marks]

(e) Find the derivative  $q'(t)$  of

$$q = \sin\left(\frac{t}{\sqrt{2-t}}\right) , \quad t < 2 .$$

Simplify your answer.

[5 marks]

(f) Find the total area of the region between the  $x$ -axis and the graph of

$$y = x^{1/5} , \quad -1 \leq x \leq 1 .$$

[5 marks]

(g) Evaluate

$$\int_2^3 \frac{6x}{3x^2 + 5} dx .$$

Simplify your answer.

[5 marks]

(h) Evaluate

$$\int x^2 e^{5x} dx$$

for  $x \in \mathbb{R}$ . Simplify your answer.

[5 marks]

**Question 2** [total: 25 marks] Consider the curve  $y = f(x)$  for the function

$$f(x) = -5x^2 + \frac{1}{4}x^4.$$

- (a) Identify the domain of  $f$  and any symmetries the curve may have. [3 marks]
- (b) Find  $f'(x)$  and  $f''(x)$ . [2 marks]
- (c) Find the critical points of  $f$ , and identify the function's behaviour at each one. [4 marks]
- (d) Find where the curve is increasing and where it is decreasing. [4 marks]
- (e) Find the points of inflection, if any occur, and determine the concavity of the curve. [4 marks]
- (f) Identify any asymptotes. [1 marks]
- (g) Plot key points, such as intercepts, critical points, and points of inflection, and sketch the curve. [7 marks]

**Question 3** [total: 20 marks]

- (a) State the definition of the derivative of the function  $f(x)$  with respect to the variable  $x$ . [5 marks]
- (b) By differentiating from first principles, show that the derivative of  $f(x) = \sqrt{x}$  does not exist at  $x = 0$ . [5 marks]
- (c) By differentiating from first principles, show that the two one-sided derivatives of  $f(x) = |x|$  are not equal at  $x = 0$  and hence that the derivative does not exist. [10 marks]

**Question 4** [15 marks] The sum of two non-negative numbers is 20. Find the numbers if one number plus the square root of the other is to be as large as possible.

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**End of Paper**