

Long range dependence, fractional renewal models, and Bayesian inference

N.W. Watkins¹, T. Graves², C. Franzke³, R. Gramacy⁴, E. Tindale⁵

¹LSE, U Warwick and Open University, UK

²Arup, UK

³U Hamburg, Germany

⁴Virginia Tech, USA

⁵U Warwick, UK

This timely session is testimony to the fact that the importance of heavy tailed probability distributions, including the kappa distribution, is now well appreciated. A complementary problem is the effect of long range dependence (LRD) in time, and, since the 1960s, LRD as embodied by the fractional Gaussian noise and ARFIMA models has been a well-studied mechanism for the origin of $1/f$ noise and the Hurst effect. This talk will discuss two avenues of research, drawing on the authors' recent papers [Graves et al, Systematic inference of the long-range dependence and heavy-tail distribution parameters of ARFIMA models, *Physica A*, 2017; Watkins, "Mandelbrot's $1/f$ fractional renewal models of 1963-67: The non-ergodic missing link between change points and long range dependence", arXiv, 2016; Franzke et al, A Dynamical Systems Explanation of the Hurst Effect and Atmospheric Low-Frequency Variability, *Scientific Reports*, 2015; Graves et al, Efficient Bayesian inference for natural time series using ARFIMA processes, *NPG*, 2015; Graves et al, "A brief history of long memory", arXiv, 2014].

The first avenue of research concerns breakpoints. These have long been known to be an alternative to the long-range dependent kernel in fGn as a source of the Hurst effect, but recent research by one of us has shown that Mandelbrot had proposed a model with a heavy tailed distribution of time intervals between the breaks as early as 1963. By 1965-67 he was showing how this was an alternative non-ergodic model for $1/f$ noise, with consequences for model choice and time series interpretation that are increasingly becoming topical in physics and elsewhere, and are still relevant to the topic of weak ergodicity breaking.

The second concerns Bayesian inference when an LRD model is plausible. I will discuss our recent work on a novel systematic Bayesian approach for joint inference of the memory and tail parameters in an ARFIMA model with heavy-tailed innovations. I will also show its application to solar X-ray flare data.

[1] Graves et al, *Physica A* **473**, 60 (2017).

[2] Franzke et al, *Scientific Reports* **5**, 9068 (2015).

[3] Graves et al, *NPG* **22**, 679 (2015).