Microscopic Derivation of Coloured Lévy Flights in Active Swimmers' Suspensions

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The motion of a tracer particle in a complex medium typically exhibits anomalous diffusive patterns, characterised, e.g, by a non-liner mean-squared displacement and/or non-Gaussian statistics. Modeling such fluctuating dynamics is in general a challenging task, that provides, despite all, a fundamental tool to probe the rheological properties of the environment. A prominent example is the dynamics of a tracer in a suspension of swimming microorganisms, like bacteria, which is driven by the hydrodynamic fields generated by the active swimmers. For dilute systems, several experiments confirmed the existence of non-Gaussian fat tails in the displacement distribution of the probe particle, that has been recently shown to fit well a truncated Lévy distribution. This result was obtained by applying an argument first proposed by Holtsmark in the context of gravitation: the force acting on the tracer is the superposition of the hydrodynamic fields of spatially random distributed swimmers. This theory, however, does not clarify the stochastic dynamics of the tracer, nor it predicts the non monotonic behaviour of the non-Gaussian parameter of the displacement distribution. Here we derive the Langevin description of the stochastic motion of the tracer from microscopic dynamics using tools from kinetic theory. The random driving force in the equation of motion is a coloured Lévy Poisson process, that induces power-law distributed position displacements. This theory predicts a novel transition of their characteristic exponents at different timescales. For short ones, the Holtzmark-type scaling exponent is recovered; for intermediate ones, it is larger. Consistently with previous works, for even longer ones the truncation appears and the distribution converge to a Gaussian. Our approach allows to employ well established functional methods to characterize the displacement statistics and correlations of the tracer. In particular, it qualitatively reproduces the non monotonic behaviour of the non-Gaussian parameter measured in recent experiments.