

From Rocket Science to Anomalous Time Series: Concepts, history, applications and inference

The 20th century saw a revolution in science and technology, when it became possible to accurately model many aspects of real-world complexity by harnessing the power of random processes. The theory of diffusion and Brownian motion due to Einstein, Bachelier, Wiener and others is a particularly visible example where the introduction of stochastic models enhanced (rather than diminished, as is sometimes feared) physical understanding. A very major contribution was made by Langevin in 1908 when he wrote down an equation generalising Newton's second law to Brownian motion. He exploited a key approximation, the separation of time scales, which assumes that the types of complexity one needed to model were either few, and slowly varying, deterministic degrees of freedom, like Newton's falling apple, or many, but random and "fast", like an ideal gas.

This idea has proved remarkably powerful in practise, and demanding applications including (metaphorical) "rocket science", the Black-Scholes model of stock prices, and real rocket science, the Kalman filter so central to guidance systems, have driven many advances. Progress has required blending insights and tools from the fields of stochastic processes, statistical physics and statistical inference, and now forms a very mature body of knowledge [1].

Despite being remarkably durable, Langevin's approximation isn't always good enough, however. Several important real world systems are much more complex. Some violate Langevin's approximation by having important dynamics on intermediate timescales. I will summarise my experience of the challenges posed by three cases: the interaction between control system and plasma in a Tokamak fusion reactor [2], the possibility of long-ranged response to perturbations in global climate [3], and the release of energy by the coupled solar wind-magnetosphere system in "space weather" events [4].

I will also highlight two areas where we have recently made progress. One is conceptual and historical; helping to distinguish between different sources of the measured $1/f$ power spectra and "long memory" which so often signal the breaking of Langevin's approximation [5]. The other is technical; bringing to bear the power of Bayesian statistics on time series models of the ARFIMA class, which flexibly describe both long memory and also heavy tails in fluctuation amplitudes. I will show examples of Bayesian inference on solar flare [6] and climate [7] datasets.

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[2] Chapman et al, Nuclear Fusion, 2017. DOI: 10.1088/0029-5515/57/2/022017

[3] Watkins, in Franzke & O'Kane (eds.), "Nonlinear and Stochastic Climate Dynamics", 2016. DOI: 10.1017/9781316339251

[4] Watkins et al, Space Science Reviews, 2016. DOI: 10.1007/s11214-015-0155-x; Freeman and Watkins, Science, 2002. DOI: 10.1126/science.1075555

[5] Watkins, arXiv:1603.00738v1[stat.OT], 2016; Graves et al, submitted, Entropy, 2017. DOI: 10.20944/preprints201705.0194.v1

[6] Graves et al, Physica A, 2017. DOI: 10.1016/j.physa.2017.01.028

[7] Graves et al, NPG, 2015. DOI: 10.5194/npg-22-679-2015