Power spectral densities for random motion in presence of strong frozen disorder

Gleb Oshanin

Laboratoire de Physique Théorique de la Matière Condensée (UMR CNRS 7600) Sorbonne Universités - UPMC, Paris, France

In this talk I overview our recent results on power spectral densities of diffusive processes in two models of disordered systems – Sinai model with *periodic* disorder and passive advection in layered random flows (Matheron -- de Marsily model).

In the first part, we consider a system in which the potential is given by a Brownian bridge on a finite interval (0,L) and then periodically repeated over the whole real line and study the standardly defined power spectrum S(f) of the diffusive process x(t) in such a potential [1]. We show that for most of realizations of x(t) in a given realization of the potential, the low-frequency behavior is S(f) ~ A/f², i.e., is the same as for standard Brownian motion, and the amplitude A is a disorder-dependent random variable with a finite support. Focusing on the statistical properties of this random variable, we determine the moments of A of arbitrary, negative, or positive order k and demonstrate that they exhibit a multi-fractal dependence on k and a rather unusual dependence on the temperature and on the periodicity L, which are supported by atypical realizations of the periodic disorder. We finally show that the distribution of A has a log-normal left tail and exhibits an essential singularity close to the right edge of the support, which is related to the Lifshitz singularity.

In the second part, we consider dynamics of a particle in a two-dimensional model system with random (along the y-axis), frozen layered flows parallel to the x-axis. A particle undergoes an unbiased fractional Brownian motion y(t) with Hurst index H (0< H < 1) inbetween the layers, and is passively carried in a random direction along the x-axis until it leaves a given layer. We concentrate on the process x(t) of random passive advection and show that the mean squared displacement of x(t) grows superdiffusively, $\langle x^2(t) \rangle \approx t^{2-H}$. Next, we study the disorder averaged Wigner-Ville power spectrum W(f) of x(t) and show that W(f) exhibits a strong singularity in the limit of an infinite observation time T, W(f) $\approx 1/f^{3-H}$. Lastly, we show that the variance of W(f) with respect to disorder vanishes as T $\Rightarrow \infty$, meaning that the spectrum is self-averaging.

[1] David S. Dean, Antonio Iorio, Enzo Marinari and Gleb Oshanin, Phys. Rev. E 94, 032131 (2016)

[2] Enzo Marinari, Gleb Oshanin and Alessio Squarcini, in preparation