MTH4103 (2013–14)

Geometry I

Description and Syllabus **Winter/Spring 2014**

1 Description

Properties of two- and three-dimensional space turn up almost everywhere in mathematics. For example, vectors represent points in space, equations describe shapes in space and transformations move shapes around in spaces; a fruitful idea is to classify transformations by the points and shapes that they leave fixed. Most mathematicians like to be able to 'see' in special terms why something is true, rather than simply relying on formulas. This module ties together the most useful notions from geometry—which give the meaning of the formulas—with the algebra that gives the methods of calculation. It is an introductory module assuming nothing beyond the common core of A-level Mathematics or equivalent.

2 Syllabus

1. Phrasebook up to $\mathbb{R}^3$.
2. Vectors in 2-space and 3-space, expressed as $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ or as row or column vectors. Addition of vectors. Length of vectors.
3. Vector, parametric and Cartesian equations of a straight line in $\mathbb{R}^2$ and $\mathbb{R}^3$.
4. Scalar multiple and scalar product of vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$. Cartesian equation of a plane in $\mathbb{R}^3$. Intersections of two or three planes. Solution of families of linear equations in x, y, z by reduction to echelon form.
5. Vector products in $\mathbb{R}^3$. Volume of parallelepiped as given by triple scalar product and determinant.
6. Linear transformations of $\mathbb{R}^2$, expressed by matrices with respect to the standard basis $\mathbf{i}$, $\mathbf{j}$. Examples: rotations, reflections, dilations, shears; their matrices.
7. For 2×2 matrices: characteristic equation, eigenvalues and eigenvectors, trace. Application to the examples in (6) (e.g. rotations with integer trace and the crystallographic restriction).
8. Extension of (6), (7) to 3×3 matrices.
9. Addition and multiplication of arbitrary sized matrices (where defined). Their interpretation as addition and composition of linear transformations. Inversion of 2×2 and 3×3 matrices.