Geometry I, 2010 : Week-12 test

Last name:

First name:

Student number:

The duration of this test is **40 minutes**. Answer **all** 10 questions. Each question is worth 1 mark. Only the final answer to a question will be marked, so indicate this answer clearly. Calculators are **not** allowed.

Answer all questions in the spaces provided. You may do additional rough work on the backs of the question sheets, but this will not be looked at.

1. Let $A = \begin{pmatrix} 2 & 3 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & -1 \\ 3 & 4 \end{pmatrix}$. Determine the matrix $2A-B$.

2. Let $A = \begin{pmatrix} 2 & 3 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -2 \\ -2 & 4 \end{pmatrix}$. Determine the matrix AB.

3. Write down the inverse of the 2×2 matrix $A = \begin{pmatrix} 2 & 5 \\ 2 & 4 \end{pmatrix}$. [Simplify your matrix entries as much as possible.]

4. Calculate the determinant of $A = \begin{pmatrix} 7 & 5 & 1 \\ -2 & 3 & 1 \\ -2 & 4 & 1 \end{pmatrix}$.

5. Write down the matrix representing the linear transformation $t : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $t\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x - y \\ 2z + x \end{pmatrix}$.

6. Determine the 2×2 matrix representing a rotation about the origin through an angle of $3\pi/2$ (anticlockwise). [Simplify your matrix entries as much as possible.]

7. Determine all the (real) eigenvalues of the matrix $\begin{pmatrix} 8 & -6 \\ 9 & -7 \end{pmatrix}$.

8. Let $A = \begin{pmatrix} -1 & 1 \\ 4 & -1 \end{pmatrix}$. Determine the set of all eigenvectors of A which correspond to the eigenvalue 1. [Your answer must be given correctly as a set.]

9. Exactly which of the following statements are true? [Your answer must be completely correct to obtain a mark.]

(a) Whenever A and B are invertible 2×2 matrices then $A - B$ is also an invertible 2×2 matrix.

(b) If A and B are 2×2 matrices and AB is the zero 2×2 matrix, then either A or B is the zero 2×2 matrix.

(c) If the second column of a 3×3 matrix A is a scalar multiple of the first column, then $\det(A) = 0$.

(d) If A and B are invertible 3×3 matrices, then $(AB)^{-1} = A^{-1}B^{-1}$.

(e) If $t : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation, then $t(\mathbf{0}_3) = \mathbf{0}_2$.

10. Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Find all 2×2 matrices $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $AB = BA$. [Give your answer as a set.]