Geometry I, 2009 : Week-12 test

Last name:

First name:

Student number:

The duration of this test is **40 minutes**. Answer **all** 10 questions. Each question is worth 1 mark. Only the final answer to a question will be marked, so indicate this answer clearly. Calculators are **not** allowed.

Answer all questions in the spaces provided. You may do additional rough work on the backs of the question sheets, but this will not be looked at.

1. Let $A = \begin{pmatrix} -1 & 2 \\ 3 & -2 \end{pmatrix}$. Determine the matrix $-2A + 3I_2$.

2. Let $A = \begin{pmatrix} -1 & 2 \\ 3 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -2 \\ 1 & 4 \end{pmatrix}$. Determine the matrix AB.

3. Suppose A is an $m \times n$ matrix and B is a $p \times q$ matrix. State the only condition(s) on m, n, p, q required to be able to form the product AB.

4. Calculate the determinant of $A = \begin{pmatrix} -2 & 1 & -1 \\ -3 & 1 & 2 \\ 1 & -4 & 3 \end{pmatrix}$.

5. Determine the 2×2 matrix representing a rotation about the origin through a counterclockwise angle of $\pi/2$. [Simplify your matrix entries as much as possible.]

6. Determine the 2×2 matrix representing the transformation made by performing a rotation about the origin through a counterclockwise angle of $3\pi/2$ followed by a reflection in the line through the origin at counterclockwise angle $\pi/4$ from the x-axis. [Simplify your matrix entries as much as possible.]

7. Determine the characteristic polynomial of the matrix $\begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix}$.

8. Let $A = \begin{pmatrix} -3 & 1 \\ 6 & -2 \end{pmatrix}$. Determine the set of all eigenvectors of A with corresponding eigenvalue 0. [Your answer must be given correctly as a set.]

9. Exactly which of the following statements are true? [Your answer must be completely correct to obtain a mark.]

(a) Whenever A is an invertible 2×2 matrix then αA is also invertible for all scalars α.

(b) $\det(AB) = \det(BA)$ for all 2×2 matrices A and B.

(c) For all invertible 2×2 matrices A, we have $\det(A^{-1}) \neq \det(A)$.

(d) $\det(-A) = -\det(A)$, for all 3×3 matrices A.

(e) $A(BC) = (AB)C$ for all $n \times n$ matrices A, B, C.

10. Determine all the 2×2 matrices which have all entries 0 or 1 and which do not have zero as an eigenvalue. [For example, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is one such matrix.]