MTH4103: Geometry I

Queen Mary
University of London

Mid-term Test 2010

Duration: 40 minutes

Date and Time: 12th November 2010, in the hour 12:30–13:30

Last name: ______________________________

First name(s): ______________________________

Student number: ______________________________

The duration of this test is **40 minutes**. Answer **all** 10 questions. Each question is worth an equal number of marks (1 each). Apart from Question 5, only the final answer to a question will be marked, so indicate this answer clearly. Simplify your answers as much as possible.

Calculators are **not** permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Answer all questions in the spaces provided. You may do additional rough work on the backs of the question sheets or at the end, but this will not be looked at.

Do **not** turn over until an invigilator instructs you to do so.

For Examiner's use only

Question	Mark	Question	Mark
1		6	
2		7	
3		8	
4		9	
5		10	
		Total (%)	

Question 1. Let $\mathbf{a} = \begin{pmatrix} -3 \\ -2 \\ 6 \end{pmatrix}$. Determine $|\mathbf{a}|$.

Question 2. Let $A = (-3, 3, -2)$ and $B = (-3, 4, 0)$, and let P be one third of the way along the line segment from A to B. (That is $|\overrightarrow{AP}| = \frac{1}{3}|\overrightarrow{AB}|$.) Determine the position vector $\mathbf{p}$ of P.

Question 3. Determine parametric equations for the line through the points $(-2, 5, 0)$ and $(-3, 3, -2)$.

Question 4. Determine the cosine of the angle between the vectors $\mathbf{p} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$. Remember to simplify your answer as much as possible.

Question 5. Apply Gaußian elimination to transform the following system of linear equations into echelon form. [You do not need to determine the solutions of the system.]

$$\left.\begin{aligned} y + 5z &= -5 \\ x - 2y + 5z &= -5 \\ -2x + 4y + 5z &= -5 \end{aligned}\right\} .$$

Question 6. Determine the intersection, as a set of points, of the plane defined by the equation $x - 2y + 2z = 3$ with the plane defined by $-y + 3z = 2$. [Your answer must be correctly specified as a set to obtain a mark.]

Question 7. Calculate the area of the parallelogram determined by the vectors $\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \\ -3 \end{pmatrix}$.

Question 8. Determine a nonzero vector parallel to the plane defined by

$$3x + y - 3z = 17.$$

Question 9. Exactly which of the following statements are true? [Ambiguous answers will score zero, and your answer must be completely correct to obtain a mark.]

(a) If $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ are vectors such that $\mathbf{u}$ is orthogonal to $\mathbf{v}$ and $\mathbf{v}$ is orthogonal to $\mathbf{w}$, then $\mathbf{u}$ is orthogonal to $\mathbf{w}$.

(b) $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ implies that $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

(c) $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ for all vectors $\mathbf{v}$ implies that $\mathbf{u} = \mathbf{0}$.

(d) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$ for all vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$.

(e) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) \neq (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ for all vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$.

Question 10. Let A, B, C be points in 3-space with position vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ respectively, and let D be the point of 3-space such that $ABCD$ is a parallelogram.

Determine an expression for the position vector $\mathbf{d}$ of D in terms of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$.

For rough work only.